42 YEARS OF EXCELLENCE

NARAYANA EDUCATIONAL INSTITUTIONS

NARAYANA'S SENSATIONAL SUCCESS ACROSS INDIA

7 Students Secured 100 Percentile in All India JEE Main-2020

BELOW 10: **21** RANKS (All Cat)

BELOW 100: **113** RANKS (All Cat)

M LIKHITH REDDY	SHIVA KRISHNA	TANUJA	SWAYAM CHUBE	HARSHVARDAHAN AGARWAL	YESHASH CHANDRA	PARTH DWIVEDI
HT No. TL0305724	HT No. TL01330751	HT No. TL01327005	HT No. MR16303069	HT No. HR09300789	HT No. TL02305081	Appl. No. 200310166791

ADMISSIONS OPEN (2020-21)

OUR REGULAR CLASSROOM PROGRAMME

One Year Classroom Program
JEE/NEET-2021
(for students moving from XI to XII)

Two Year Classroom Program
JEE/NEET-2022
(for students moving from X to XI)

Three Year Integrated Classroom Program
JEE/NEET-2023
(for students moving from IX to X)

Four Year Integrated Classroom Program
JEE/NEET-2024
(for students moving from VIII to IX)

FOUNDATION PROGRAMMES
For NTSE, NSEJS, JSTSE, Olympiads & School/Board Exams
(for students moving to Class VI, VII, VIII, IX & X)

APEX BATCH
Two years school Integrated Classroom Program - 2022
For JEE Main & Advance / NEET (for XI Studying Students)
Course Feature •Complete Coverage of CBSE •Regular Classes •Weekly Test & Regular Analysis •Lab Facility •Motivation & Counseling •Competetive Exam Prep. •Ample time for self study

Online Classes for IIT/NEET/Foundation/Olympiads

- Access Recording of Past Classes on n-Learn App
- Online Parent Teacher Meeting
- Personalized Extra Classes & Live Doubt Solving
- Hybrid/Customized Classroom model
- Video Solution of Weekly/Fortnightly Test
- Printed Study Material will be sent by us
- n-Lean App
- Counselling Motivational sessions
- Affordable Fee
- Doubt Classes / Practice Classes
- Provision to Convert from online to regular classroom programme
- Once Classes resume by just paying nominal fee

Online Test

- Micro & Macro Analysis
- Relative performance (All India Ranking)
- Question wise Analysis
- Unlimited Practice Test
- Grand Test

NARAYANA Digital Classes
STUDY ONLINE FROM HOME

For Class 7th to 12th+

TOLL FREE : 1800 102 3344 | Visit us at : www.narayanagroup.com

THE NARAYANA GROUP

JEE-MAIN-2021
FEBRUARY ATTEMPT
26.02.21_SHIFT-I
MATHEMATICS

1. $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx =$

a*) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{8}$ d) $\frac{\pi}{3}$

Sol: $I = \int_0^{\pi/2} \left(\frac{\cos^2 x}{1+3^x} + \frac{\cos^2 x}{1+3^{-x}} \right) dx = \int_0^{\pi/2} \left(\frac{\cos^2 x}{1+3^x} + \frac{3^x \cos^2 x}{1+3^x} \right) dx = \int_0^{\pi/2} \cos^2 x \, dx$

$= \frac{1}{2}\int_0^{\pi/2} (1+\cos 2x) dx = \frac{1}{2}\left(x + \frac{1}{2}\sin 2x \right)\Big|_0^{\pi/2} = \frac{1}{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$

2. Value of $\lim_{x\to 0} 2\left\{ \frac{\sqrt{3}\sin\left(\frac{\pi}{6}+x\right) - \cos\left(\frac{\pi}{6}+x\right)}{\sqrt{3}x\left(\sqrt{3}\cos x - \sin x\right)} \right\}$ is equal to

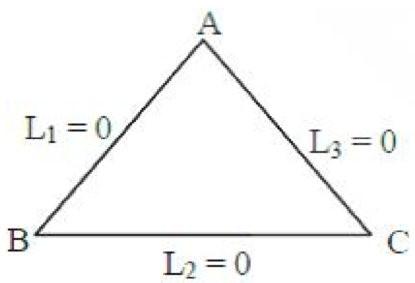
*a) $\frac{4}{3}$ b) $\frac{2}{\sqrt{3}}$ c) $\frac{2}{3}$ d) $\frac{4}{\sqrt{3}}$

Sol: $= \lim_{x\to 0} \frac{2\left[\sin\left(\frac{\pi}{6} + x - \frac{\pi}{6} \right) \right]}{\sqrt{3}x\left(\sqrt{3}\right)} == \lim_{x\to 0} \frac{4}{3} \frac{\sin x}{x} = \frac{4}{3}$

3. $x - y = 0,\ 2x + y = 6,\ x + 2y = 3$ are three line forming a triangle, then the triangle is

*a) Isosceles b) Right angle c) equilateral d) None of these

Sol:



$L_1 : x - y = 0$

$L_2 : x - 2y = 3$

$L_3 : 2x + y = 6$

$A(2,2)$

$B(1,1)$

$C(3,0)$

$\Rightarrow AB = \sqrt{2},\ BC = \sqrt{5},\ AC = \sqrt{5}$

$\therefore$ Triangle is isosceles

4. Find the number of integral values of k for which the equation 3 sin x + 4 cos x = k + 1 has solution.

a) 13 b) 6 c) 8 *d) 11

sol: $-\sqrt{3^2 + 4^2} \le 3\sin x + 4\cos x \le \sqrt{3^2 + 4^2}$

$-5 \le (k+1) \le 5$

$-6 \le k \le 4$

5. Number of 7 digits number in which sum of digits is 10 and digits can take 1, 2, 3 values, is

*a) 77 b) 42 c) 60 d) 35

sol: Cases-1: 1, 1, 1, 1, 1, 2, 3

ways = $\dfrac{7!}{5!} = 42$

Case = 2 : 1, 1, 1, 2, 2, 2

ways = $\dfrac{7!}{4!\,3!} = 35$

total ways = 42 + 35 = 77

6. Find the number of solution of the equation $4(x - 1) = \log_2 (x - 3)$

*a) 0 b) 42 c) 60 d) 35

sol; $4(x-1) = \log_2(x-3)$

$2^{4(x-1)} = (x-3)$ here $x \geq 3$

So no solution

7. If A is a symmetric matrix of order 2 and sum of diagonal elements of A^2 is 1, where elements of matrix are integer, then number of such matrices are

*a) 4 b) 6 c) 8 d) 5

sol; $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$A^2 = \begin{bmatrix} a^2+b^2 & b(a+c) \\ b(a+c) & b^2+c^2 \end{bmatrix}$

$tr(A^2) = a^2 + 2b^2 + c^2 = 1$

$\Rightarrow b = 0$ and $a^2 + c^2 = 1$

$\Rightarrow (a,c) \equiv (1,0),(-1,0),(0,-1)$

8. The maximum value of slope of tangent to $y = \frac{x^4}{2} - 5x^3 + 18x^2 + 6$ is at a point

a) $(2, 2)$ *b) $(2, 46)$ c) $\left(1, \frac{39}{2}\right)$ d) $(1, 0)$

sol; $m = \frac{dy}{dx} = 2x^3 - 15x^2 + 36x$

$\frac{dy}{dx} = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6) = 0$

$\Rightarrow x = 2, 3$

$$\frac{d^2y}{dx^2} = 6(2x-5)$$

$$\left.\frac{d^2y}{dx^2}\right|_{n=2} = -ve$$

$\therefore$ Maximum at x = 2

Point (2, 46)

9. $\{(P,Q); P,Q$ be 2 points which are equidistant from origin$\}$, then point (x,y) which are equivalent class of $(1,-1)$

a*) $x^2+y^2=2$ b) $x^2+y^2=\sqrt{2}$ c) $x^2+y^2=1$ d) $x^2+y^2=2\sqrt{2}$

sol: The equivalence class containing (1, -1) for this relation is $x^2+y^2=2$

10. Value of $\begin{vmatrix} (a+1)(a+2) & (a+1) & 1 \\ (a+2)(a+3) & (a+2) & 1 \\ (a+3)(a+4) & (a+3) & 1 \end{vmatrix}$ is equal to

*a) -2 b) 2 c) 0 d) 1

sol; $D = \begin{vmatrix} a^2+3a+2 & a+1 & 1 \\ a^2+5a+6 & a+2 & 1 \\ a^2+7a+12 & a+3 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$D = \begin{vmatrix} a^2+3a+2 & a+1 & 1 \\ 2a^24 & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix} = 4a+8-4a-10 = -2$$

11. If $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c}$, then find the value of $\cos\left(\frac{\pi c}{a+b}\right)$

a) $\frac{1+y^2}{1-y^2}$ b) $\frac{2y}{1+y^2}$ c*) $\frac{1-y^2}{1+y^2}$ d) $\frac{y}{1+y^2}$

sol; Let $\sin^{-1}x=a\lambda,\cos^{-1}x=b\lambda,\tan^{-1}y=c\lambda$

$$\Rightarrow (a+b)\lambda=\frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{a+b}=2\lambda$$

Now $\cos\left(\frac{\pi}{a+b}\right)=\cos(2\lambda c)=\cos\left(2\tan^{-1}y\right)$

$$=\frac{1-y^2}{1+y^2}$$

12. If $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)=$

*a) $\left|\vec{a}\right|^4\vec{b}$ b) $-\left|\vec{a}\right|^4\vec{b}$ c) $\left|\vec{a}\right|^2\vec{b}$ d) $-\left|\vec{a}\right|^2\vec{b}$

sol: $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\vec{a}-\left|\vec{a}\right|^2\vec{b}\right)\right)$

$$=\vec{a}\times\left(-\left|\vec{a}\right|^2\left(\vec{a}\times\vec{b}\right)\right)=--\left|\vec{a}\right|^2\left(\left(\vec{a}.\vec{b}\right)\vec{a}-\left|\vec{a}\right|^2\vec{b}\right)=-\left|\vec{a}\right|^4\vec{b}-\left|\vec{a}\right|^2\left(\vec{a}.\vec{b}\right)\vec{a}$$

$=\left|\vec{a}\right|^4\vec{b}$ $\left(\because\ \vec{a}.\vec{b}=0\right)$

13. If $\left|f(x)-f(y)\right|\le\left|(x-y)^2\right|$; $x,y\in R$ and $f(0)=1$ then

a) $f(x)=0$ for $x\in R$ *b) $f(x)>0 : x\in R$

c) $f(x)<0 : x\in R$ d) $f(x)$ can take any value sol; $\left|\frac{f(x)-f(y)}{x-y}\right| \le |x-y|$

$\Rightarrow |f(x)| \le 0$

$\Rightarrow |f(x)| = 0$

$\Rightarrow |f(x)| = \text{cons tan t}$

$\Rightarrow |f(x)| = 1$

14. $1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}........\infty$

*a) $\frac{13}{4}$ b) $\frac{13}{2}$ c) $\frac{11}{4}$ d) $\frac{11}{2}$

sol; $S = 1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+............\infty$

$\frac{1}{3}S = \frac{1}{3}+\frac{2}{3^2}+\frac{7}{3^3}+............$ (ii)

(i) + (ii)

$\frac{2}{3}S = 1+\frac{1}{3}+\frac{5}{3^2}+\frac{5}{3^3}+......$

$\frac{2}{3}S = \frac{4}{3}+\frac{5}{3^2}+\frac{5}{3^3}+......$

$\frac{2}{3}S = \frac{4}{3}+\frac{\frac{5}{3^2}}{1-\frac{1}{3}} = \frac{4}{3}+\frac{5}{6} = \frac{13}{6}$

$S = \frac{13}{6}\times\frac{3}{2} = \frac{13}{4}$

15. Find maximum value of term independent of t in expansion of $\left(tx^{1/5}+\frac{(1-x)^{1/10}}{t}\right)^{10}$

*a) $56\sqrt{3}$ b) $\frac{56}{\sqrt{3}}$ c) 56 d) $28\sqrt{3}$

sol; $T_{r+1}={}^{10}C_r\left(t\,x^{1/5}\right)^{10-r}\left(\frac{(1-x)^{1/10}}{t}\right)^{r}$

$10-r-r=0\Rightarrow r=5$

$T_6={}^{10}C_5x(1-x)^{1/2}$

$\frac{d(T_6)}{dx}={}^{10}C_5\left((1-x)^{1/2}+\frac{-x}{2\sqrt{1-x}}\right)=0$

$2(1-x)-x=0\Rightarrow x=\frac{2}{3}$

$T_6={}^{10}C_3\frac{2}{3}\left(\frac{1}{3}\right)^{1/2}=56\sqrt{3}$

16. $\sum_{n=1}^{n=100}\int_{n-1}^{n}e^{x-[x]}dx$ is equal to

*a) 100 (e-1) b) 100 e c) 100 d) 100 (1 - e)

sol; $\sum_{n=1}^{n=100}\int_{n-1}^{n}e^{\{x\}}$

$=100\int_0^1 e^x=100(e-1)$

17. If a fair coin is tossed n times, probability of getting 9 heads, is equal to probability of getting 7 heads. Find the probability of getting 2 heads.

*a) ${}^{16}C_2 \times \left(\frac{1}{2}\right)^{16}$ b) ${}^{16}C_2 \times \left(\frac{1}{2}\right)^{14}$

c) ${}^{16}C_3 \times \left(\frac{1}{2}\right)^{16}$ d) ${}^{16}C_3 \times \left(\frac{1}{2}\right)^{14}$

sol; ${}^{n}C_9 \times \left(\frac{1}{2}\right)^{9} \times \left(\frac{1}{2}\right)^{n-9} = {}^{n}C_7 \times \left(\frac{1}{2}\right)^{7} \times \left(\frac{1}{2}\right)^{n-7}$

${}^{n}C_9 \times {}^{n}C_7 \Rightarrow n = 16$

P(2 Heads) = ${}^{16}C_2 \left(\frac{1}{2}\right)^{2} \times \left(\frac{1}{2}\right)^{14}$

$= {}^{16}C_2 \times \left(\frac{1}{2}\right)^{16}$

18. Given three planes $\begin{array}{l} P_1 : 3x - 15y + 21z = 9 \\ P_2 : 4x - 20y + 21z = 10 \\ P_3 : 2x - 10y + 14z = 10 \end{array}$

then

a) P_1, P_2 are parallel b) P_1, P_2, P_3 are parallel

*c) P_1, P_3 are parallel d) P_2, P_3 are parallel

Sol:

$P_1 : x - 5y + 7z = 3$

$P_2 : 4x - 20y + 21z = 10$

$P_3 : x - 5y + 7z = 8$

P_1 and P_3 are parallel as dr's of normal are same

19. The summation of 2nd & 6th terms of an increasing GP is $\frac{25}{2}$ and product of 3rd & 5th term is 25, then summation of 4th, 6th & 8th term is

a) 30 *b) 35 c) 20 d) 22

Sol: $ar + ar^5 = \frac{25}{2}$ and $ar^2 . ar^4 = 25$

$\therefore \frac{r + r^5}{r^3} = \frac{5}{2}$

$\Rightarrow 2 + 2r^4 = 5r^2$

$\Rightarrow 2r^4 - 5r^2 + 2 = 0$

$\Rightarrow r^2 = 2$ or $r^2 = \frac{1}{2}$ reject

Now, $ar^3 + ar^5 + ar^7 = 5 + ar^5\left(1 + r^2\right) = 5 + 5.2(1 + 2) = 35$

20. If $P(1,5,35)\ Q(7,5,2)\ R(1,\lambda,7)\ S(2\lambda,1,2)$ are coplanar then sum of value of λ is

a) $\frac{39}{5}$ *b) $\frac{17}{2}$ c) $\frac{-39}{5}$ d) $\frac{-17}{2}$

sol: For points to be coplanar $\begin{vmatrix} 6 & 0 & -33 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -38 \end{vmatrix} = 0$

$\Rightarrow 6(-33\lambda+165-112)+33(2\lambda^2-11\lambda+5)=0$

$\Rightarrow -198\lambda+318+66\lambda^2-363\lambda+165=0$

$\Rightarrow 66\lambda^2-561\lambda+483=0$

$\text{sum}=\frac{561}{66}=\frac{187}{22}=\frac{17}{2}$

21. $\int_0^{\pi}|\sin 2x|\,dx=?$

Ans: 2

Sol: $\int_0^{\pi}|\sin 2x|\,dx$

Here $f(2a-x)f(x)$

$=2\int_0^{\pi/2}(\sin 2x)dx$

$=2\left(-\frac{\cos 2x}{2}\right)_0^{\pi/2}$

$=2$

22. If $30.\,{}^{30}C_0 + 29.\,{}^{30}C_1 + 28.\,{}^{30}C_2 + ... + {}^{30}C_{29} = n.2^m$ then find the value of (m + n)

Ans: 59

Sol: General term = (30 - r). ${}^{30}C_r$

L.H.S. = $\sum_{r=0}^{30}(30-r).\,{}^{30}C_r$

$= 30\sum_{r=0}^{30}{}^{30}C_r - \sum_{r=0}^{30} r.\,{}^{30}C_r$

$= 30.2^{30} - 30.2^{29}$

$= 30.2^{29}$

So n = 30, m = 29

m +m n = 59

23. If $x^3 - 2x^2 + 2x - 1 = 0$ has roots α, β, γ then find $\left(\alpha^{162} + \beta^{162} + \gamma^{162}\right)$

Ans: 3

Sol: $n = 1,\ n = -\omega, n = -\omega^2$

$\alpha = 1,\ \beta = -\omega, \gamma = -\omega^2$

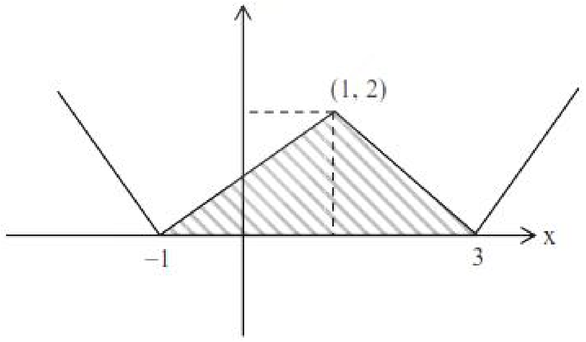
$E = 1 + \omega^{162} + \left(\omega^2\right)^{162}$

= 3

24. Find the area bounded by the curve $y = ||x-1|-2|$ with x-axis

Ans: 4

sol:



$$\text{Area} = \frac{1}{2}\times 4\times 2 = 4$$

25. Find number of solutions of $\sqrt{3}\cos^2 x = \left(\sqrt{3}-1\right)\cos x + 1$ in $x \in \left[0, \frac{\pi}{2}\right]$

Ans: 1

sol; $\sqrt{3}\cos^2 x - \left(\sqrt{3}-1\right)\cos x - 1 = 0$

$$\cos x = \frac{\left(\sqrt{3}-1\right) \pm \sqrt{\left(\sqrt{3}-2\right)^2 + 4\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{\left(\sqrt{3}-1\right) \pm \sqrt{\left(\sqrt{3}-2\sqrt{3}\right)}}{2\sqrt{3}} = \frac{\left(\sqrt{3}-1\right) \pm \left(\sqrt{3}+1\right)}{2\sqrt{3}}$$

$$= 1, \frac{-1}{\sqrt{3}}$$

since $x \in \left[0, \frac{\pi}{2}\right]$

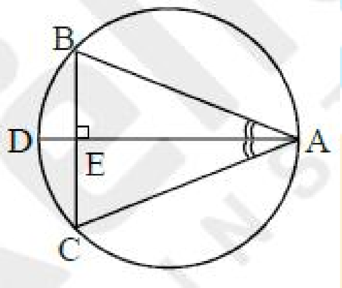
$\Rightarrow \cos x = \frac{-1}{\sqrt{3}}$, not possible

$\therefore \cos x = 1$

$\Rightarrow x = 0$

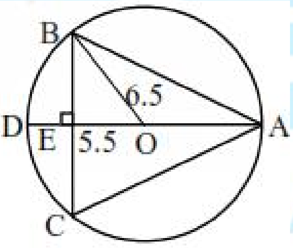
$\therefore$ number of solution 1

26. In the given figure AD = 13, DE = 1, AD bisects angle BAC and BC is perpendicular to AD, then, area of triangle ABC.



Ans: 41.568

sol: Let O be mid-point of AD, now perpendicular from C to BC bisects chord BC, $(\Delta ACE \text{ and } \Delta ABE)$ are congruent). Hence AD is diameter and O is centre of circle.



So $Be = \sqrt{(6.5)^2 - (5.5)^2}$

$= \sqrt{12}$

Hence area = $\frac{1}{2}.12.2\sqrt{12} = 24\sqrt{3}$

27. Find the difference between the value of degree and order of differential equation corresponding to the family of curves $y^2 = a\left(x+\sqrt{2}\right)$

Ans: 2

sol; order of differential equation is 1.

2yy' = a

$\Rightarrow y^2 = 2yy'\left(x+\sqrt{2yy'}\right)$

$\Rightarrow y-2xy' = 2y'.\sqrt{2yy'}$

$\Rightarrow (y-2xy')^2 = 4(y')^2.\sqrt{2yy'}$

$\Rightarrow \left(y-2x\frac{dy}{dx}\right)^2 = 8y.\left(\frac{dy}{dx}\right)^3$

Degree of differential equation = 3

28. A plane is passing through $(\lambda, 2, 1) \& (4, -2, 2)$. It is perpendicular to line joining points

$A(-2, 23, 18)$ and $B(-1, 29, 16)$. Find value of $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$

Ans: 8

sol: $\overrightarrow{AB} = \hat{i} + 6\hat{j} - 2\hat{k}$

$\alpha = (\lambda - 4)\hat{i} + 4\hat{j} - \hat{k}$

$\overrightarrow{AB} = \alpha = 0$

$\lambda - 4 + 24 + 2 = 0 \quad \Rightarrow \lambda = -22$

$E = 4 + 8 - 4 = 8$

29. Number of bacteria are increasing at a rate proportional to its number at time 't' of at $t = 0$, $N = 1000$ and after 2 hours, number of bacterial increased by 20 %. If at $t = \frac{k}{\ln\frac{5}{6}}$, number of bacteria are 2000, then find $\left(\frac{k}{\ln 2}\right)^2$?

Ans: 4

sol: $\frac{dx}{dt} \infty\ x$

$\Rightarrow \frac{dx}{dt} = \lambda x$

$\Rightarrow \int_{1000}^{x} \frac{dx}{x} = \lambda \int_{0}^{t} dt$

$\Rightarrow \ln\frac{x}{1000} = \lambda t$

at $t = 2$, $x = 1200$

$\therefore 2\lambda = \ln\frac{6}{5}$

$\therefore x = 1000.e^{\frac{1}{2}\ln\frac{6}{5}.t}$

Now $2000 = 1000.e^{\frac{1}{2}\ln\frac{6}{5}.\frac{k}{\ln\frac{5}{6}}}$

$\Rightarrow 2 = e^{\frac{k}{2}}$

$\Rightarrow \frac{k}{2} = \ln 2$

$\Rightarrow \frac{k}{\ln 2} = -2$