NARAYANA IIT ACADEMY INDIA

JEE (ADVANCED) 2019 PAPER 1

PART-I PHYSICS

SECTION 1 (Maximum Marks:12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

Q.1. Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass *m* moving in circular orbits about the common center with the same kinetic energy *K*. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r)=\rho(r)/m$ is [*G* is universal gravitational constant]

(A) $\dfrac{K}{2\pi r^2 m^2 G}$ (B) $\dfrac{K}{\pi r^2 m^2 G}$

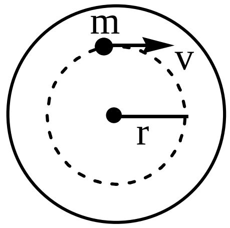
(C) $\dfrac{3K}{\pi r^2 m^2 G}$ (D) $\dfrac{K}{6\pi r^2 m^2 G}$

KEY: (A)

Sol: M(r) = mass of gas in a sphere of radius r.

Can any particle, $F_g = F_{cp}$

$$\frac{GM(r)m}{r^2}=\frac{mv^2}{r}$$



GM (r) m = 2(K) r

$$\Rightarrow M(r) = \frac{2K}{Gm} r$$

On differentiating w.r.t r, $\frac{dM}{dr} = \frac{2K}{Gm}$ .... (1)

Here dM = mass of gas in a spherical shell of radius r and thickness dr.

$$\therefore dM = [\rho(r)](dv) = [\rho(r)] 4\pi r^2 dr \text{ .... (2)}$$

Substitute (2) in (1)

$$\frac{[\rho(r)] 4\pi r^2 dr}{dr} = \frac{2K}{Gm}$$

$$\therefore \rho(r) = \frac{K}{2\pi r^2 Gm}$$

$$n(r) = \frac{\rho(r)}{m} = \frac{K}{2\pi r^2 Gm^2}$$

Q.2. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface if V_0. A hole with a small area $\alpha 4\pi R^2$ ($\alpha << 1$) is made on the shell without affecting the rest of the shell. Which one of the following statements is correct?

(A) The potential at the center of the shell is reduced by $2\alpha V_0$

(B) The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$

(C) The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}R$ from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$

(D) The magnitude of electric field at a point, located on a line passing through the hole and shell's center, on a distance $2R$ from the center of the spherical shell will be reduced by $\dfrac{\alpha V_0}{2R}$

KEY: (C)

Sol: $v_0 = \dfrac{KQ}{R} = \dfrac{K\sigma 4\pi r^2}{R}$

At center, $v_1 = v_0$ - (potential due to removed element)

$= v_0 - \dfrac{Kq}{R} = v_0 - \dfrac{K}{R}\left(\sigma\alpha 4\pi R^2\right)$

$= v_0 - v_0\alpha$

$= v_0\left(1-\alpha\right)$

Q.3. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changed with time (t) as

$$T\left(t\right) = T_0\left(1+\beta t^{\frac{1}{4}}\right),$$

where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of the metal is,

(A) $\dfrac{4P\left(T\left(t\right)-T_0\right)^3}{\beta^4 T_0^4}$ (B) $\dfrac{4P\left(T\left(t\right)-T_0\right)^4}{\beta^4 T_0^5}$

(C) $\dfrac{4P\left(T\left(t\right)-T_0\right)^2}{\beta^4 T_0^3}$ (D) $\dfrac{4P\left(T\left(t\right)-T_0\right)}{\beta^4 T_0^2}$

KEY: (A)

Sol: $P = ms\dfrac{dT}{dt}$

$= \left(H\right)\dfrac{d}{dt}\left[T_0\left(1+\beta t^{1/4}\right)\right]$

$= \dfrac{HT_0\beta}{4}t^{-3/4}$ .... (1)

$$T = T_0\left[1+\beta t^{1/4}\right]$$

$$\frac{T}{T_0} - 1 = \beta t^{1/4} \Rightarrow t^{-1/4} = \frac{\beta T_0}{T - T_0} \Rightarrow t^{-3/4} = \left(\frac{\beta T_0}{T - T_0}\right)^3 \quad \text{.............} \ (2)$$

Substitute (2) in (1)

$$P = \frac{HT_0\beta}{4}\frac{\beta^3 T_0^3}{\left(T - T_0\right)^3} \Rightarrow H = \frac{4P\left(T - T_0\right)^3}{\beta^4 T_0^4}$$

Q.4. In a radioactive sample, ${}^{40}_{19}K$ nuclei either decay into stable ${}^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable ${}^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable ${}^{40}_{18}Ca$ and ${}^{40}_{18}Ar$ nuclei are produced by the ${}^{40}_{19}K$ nuclei only. In time $t\times10^9$ years, if the ratio of the sum of stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei to the radioactive ${}^{40}_{19}K$ nuclei is 99, the value of t will be,

[Given: In 10 = 2.3]

(A) 1.15 (B) 9.2

(C) 2.3 (D) 4.6

KEY: (B)

Sol: $\lambda_{eff} = \lambda_1 + \lambda_2 = \left(4.5+0.5\right)10^{-10} = 5\times10^{-10}$ per year

$N = N_0 e^{-lt_0}$ and given that $N = \frac{N_0}{100}$

$$\frac{N_0}{100} = N_0 e^{-\lambda t_0}$$

$$10^2 = e^{\lambda t_0} \Rightarrow \frac{2\ell n10}{\lambda} = t_0$$

$$\therefore t_0 = \frac{2\times2.3}{5\times10^{-10}} = 9.2\times10^9 \text{ Years}$$

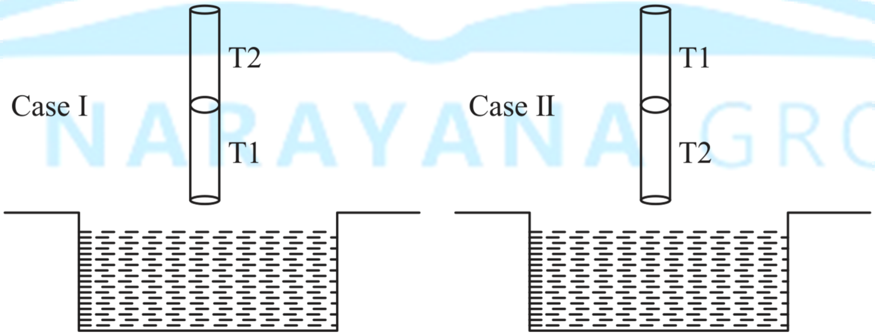
$$= t\times10^9 \text{ Years}$$

SECTION 2 (Maximum Marks:32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get -1 mark.

Q.5. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T1 and T2 of different materials having water contact angles of 0^0 and 60^0, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of following option(s) is(are) correct?
[Surface tension of water = 0.075 N/m, density of water = 1000 kg/m^3, take g = 10 m/s^2]



(A) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

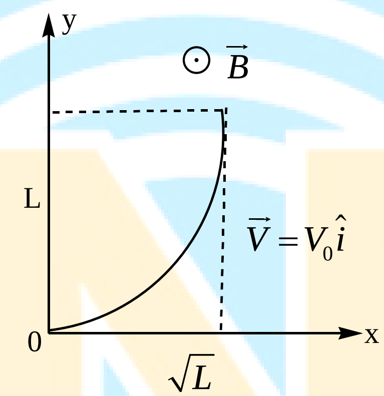
(B) For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

(C) For case I, if the joint is kept at 9 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)

(D) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)

KEY: (A, B, C)

Q.6. A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V_0, B_0, L and β are positive constants and $\Delta\Phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are :



(A) $|\Delta\Phi| = \frac{1}{2}B_0V_0L$ for $\beta = 0$

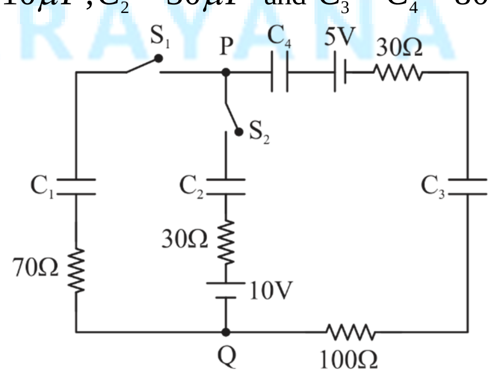
(B) $|\Delta\Phi| = \frac{4}{3}B_0V_0L$ for $\beta = 2$

(C) $|\Delta\Phi|$ remains the same if the parabolic wire is replaced by a straight wire, $y = x$ initially, of length $\sqrt{2}L$

(D) $|\Delta\Phi|$ is proportional to the length of the wire projected on the *y*-axis.

KEY: (B, C, D)

Q.7. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are $C_1 = 10\mu F, C_2 = 30\mu F$ and $C_3 = C_4 = 80\mu F$.



Which of the statement(s) is/are correct?

(A) At time $t=0$, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA.

(B) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitor C_1 will be 4 V.

(C) The key S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across 30 Ω resistor (between points P and Q) will be 0.2 A (round off to 1st decimal place).

(D) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10 V.

KEY: (A, B, C)

Q.8. A charged shell of radius R carries a total charge Q. Given Φ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct?

[$\in_0$ is the permittivity of free space]

(A) If $h>2R$ and $r>R$ then $\Phi=Q/\in_0$

(B) If $h>8R/5$ and $r=3R/5$ then $\Phi=0$

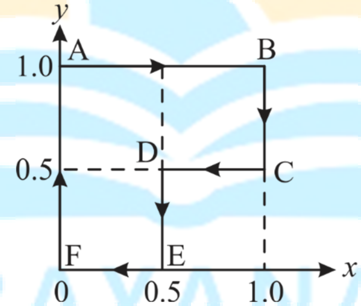
(C) If $h>2R$ and $r=3R/5$ then $\Phi=Q/5\in_0$

(D) If $h>2R$ and $r=4R/5$ then $\Phi=Q/5\in_0$

KEY: (A, B, C)

Q.9. One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature *(V-T)* diagram. The correct statement(s) is/are :

[R is the gas constant]



(A) Work done in this thermodynamic cycle $(1\to2\to3\to4\to1)$ is $|W|=\frac{1}{2}RT_0$

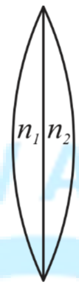
(B) The above thermodynamic cycle exhibits only isochronic and adiabatic processes.

(C) The ratio of heat transfer during processes $1\to2$ and $2\to3$ is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right|=\frac{5}{3}$

(D) The ratio of heat transfer during processes $1\to2$ and $3\to4$ is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right|=\frac{1}{2}$

KEY: (A, C)

Q.10. A thin convex lens is made of two materials with refractive indices n_1 and n_2 , as shown in figure. The radius of curvature of the left and right spherical surfaces are equal. F is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming Δn << $(n-1)$ and $1 < n < 2$, the correct statement(s) is/are,



(A) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

(B) For $n = 1.5, \Delta n = 10^{-3}$ and $f = 20cm$, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place).

(C) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

(D) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature.

KEY: (B, C, D)

Q.11. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement(s) is/are correct?

(A) The dimension of linear momentum is L^{-1}

(B) The dimension of energy is L^{-2}

(C) The dimension of force is L^{-3}

(D) The dimension of power is L^{-5}

KEY: (A, B, C)

Q.12. Two identical moving coil galvanometers have 10Ω resistance and full scale deflection at $2\mu A$ current. One of them is converted into a voltmeter of 100 mV full scale reading and the outer into an Ammeter of 1 mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with R = 1000 Ω resistor by using an idea cell. Which of the following statement(s) is/are correct?

(A) The resistance of the Voltmeter will be $100k\Omega$

(B) The resistance of the Ammeter will be 0.02Ω (round off to 2nd decimal place)

(C) The measured value of R will be $978\Omega < R < 982\Omega$

(D) If the ideal cell is replaced by a cell having internal resistance of 5 Ω then the measured value of R will be more than 1000 Ω.

KEY: (B, C)

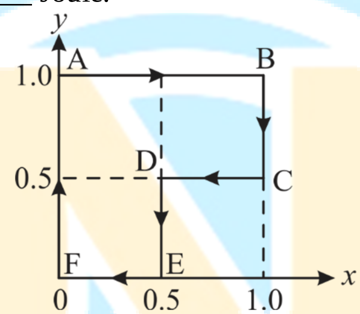
SECTION 3 (Maximum Marks:18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value of **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct option is entered;

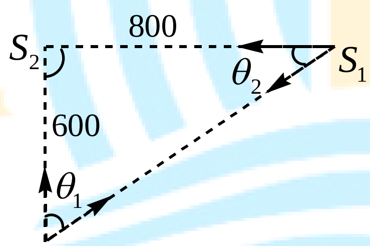
  Zero Marks : 0 In all other cases.

Q.13. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = \left(\alpha y\hat{i} + 2\alpha x\hat{j}\right)N$, where x and y are in meter and $\alpha = -1Nm^{-1}$. The work done on the particle by this force $\vec{F}$ will be ________ Joule.



KEY: (0.75)

Sol:



$$\vec{F} = -y\hat{i}2x\hat{j}$$

$AB:$

$$y = const$$

$$\vec{F} = -\hat{i} - 2x\hat{j}$$

$$d\hat{r} = dx\hat{j}$$

$$W_{AB} = \int_{x=0}^{1} \vec{F}.d\hat{r}$$

$$= \int_0^1 -dx$$

$$W_{AB} = -1$$

BC: x=const

$$\vec{F} = -y\hat{i} - 2\hat{j}$$

$$d\hat{r} = dy\hat{j}$$

$$W_{BC} = \int_{y=1}^{0.5} -2dy$$

$$W_{BC} = +1$$

CD: z const

$$\vec{F} = -0.5\hat{j} - 2x\hat{j}$$

$$d\vec{r} = dx66i$$

$$W_{CD} = \int_1^{0.5} 0.5dx$$

$$W_{CD} = 0.25J$$

DE: x = const

$$\vec{F} = -y\hat{i} - \hat{j}$$

$$\overrightarrow{dr} = dy\hat{i}$$

$$W_{DE} = \int_{y=0.5}^{0} -dy; \qquad = +0.5$$

EF:

Y = 0

$$\vec{F} = -2x\hat{j}$$

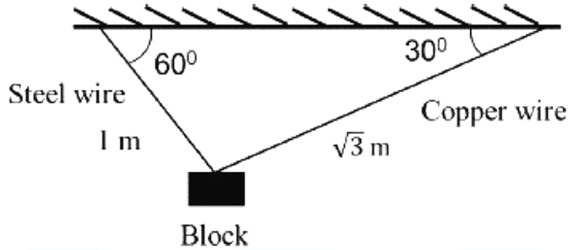
$$d\vec{r} = dx\hat{j}$$

$$W = 0$$

$$W_{ret} = -1 + 1 + 0.25 + 0.5$$

Q.14. A block of weight 100 N is suspended by copper and steel wires of same cross sectional area 0.5 cm^2 and, length $\sqrt{3}$ m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30^0 and 60^0, respectively. If elongation in copper wire is (Δl_c) and elongation in steel wire is (Δl_s), then the ratio $\frac{\Delta l_c}{\Delta l_s}$ is ________.

[Young's modulus for copper and steel are $1\times 10^{11} N/m^2$ and $2\times 10^{11} N/m^2$, respectively.]



KEY: (2)

Sol: $T_S \cos 60 = T_{cell} \cos 30$

$$\frac{T_S}{2} = T_{Cu}\frac{\sqrt{3}}{2}$$

$$T_S = \sqrt{3}T_{Cell}$$

$$T_S \sin 60 + T_{Cu}\sin 30 = 100$$

$$\sqrt{3}T_{Cu} \times \frac{\sqrt{3}}{2} + \frac{T_{Cu}}{2} = 100$$

$$2T_{Cu} = 100$$

$$T_{Cu} = 100$$

$$T_{Cu} = 50N$$

$$T_S = \sqrt{3} \times 50N$$

$\Rightarrow$ Elongation in Cu wire $\Rightarrow \frac{\Delta l_{Cu}}{l_{Cu}} = \frac{T_{Cu}}{A_r y_{cel}}$

$$\Delta l_{Cu} = l_{Cu}\frac{T_{Cu}}{Ay_{Cu}}$$

$\rightarrow$ elongation in steel wire

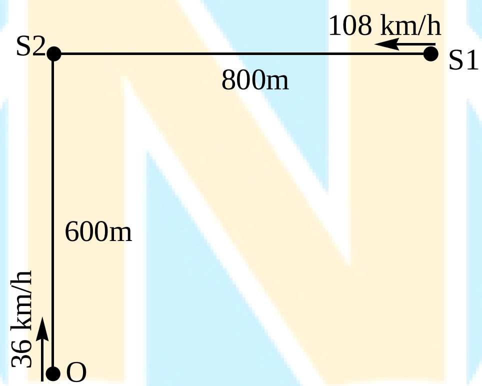
$$\Delta ls = \frac{l_s T_S}{A y_S}$$

$$\frac{\Delta l_C}{\Delta l_S} = \frac{\dfrac{l_C T_{Cu}}{A y_{Cu}}}{\dfrac{l_S T_S}{A y_{stell}}} \Rightarrow \frac{l_{Cu} T_{Cu}}{y_{Cu} l_S T_S} Y_{stell}$$

$$= \frac{\sqrt{3} \times 50 \times 2 \times 10}{1 \times 10^{11} \times 1 \times \sqrt{3} \times 50}$$

= 2

Q.15. A train S1, moving with a uniform velocity of 108 km/h, approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S2, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S2 and distance between S1 and S2 is 800 m, the number of beats heard by O is ________.
[Speed of the sound = 330 m/s]



KEY: (8.00 (7.00-10.00)*)

Sol: $VS_1 = 108 \times \frac{5}{18} = 30 m/s$

$$\tan\theta_1 = \frac{800}{600} = \frac{4}{3}$$

$$\theta_1 = 53^o$$

$$\theta_2 = 37^o$$

$\Rightarrow$ Freq. Heared form train at $S_2 = f_2$

$$f_2 = \left(\frac{V+V_0}{V}\right) fs$$

$$= \left(\frac{330+10}{330}\right) \times f_s$$

$$= \frac{340}{330} f_s = 1.030 f_s$$

$\Rightarrow$ freq. Heared from train at $S_1 = f_1$

$$f_1 = \left(\frac{V+V_0\cos\theta_0}{V-V_s\cos\theta_2}\right) f_s; \quad = \left(\frac{330+10\times\frac{3}{5}}{330-30\times\frac{4}{5}}\right) f_s$$

$$f_1 = \left(\frac{336}{306}\right) f_s = 1.098 f_s$$

$\Rightarrow$ Beat fieq $= 1.098 f_s - 1.030 f_s$

$$= 0.068 f_s$$

$$= 0.068 \times 120$$

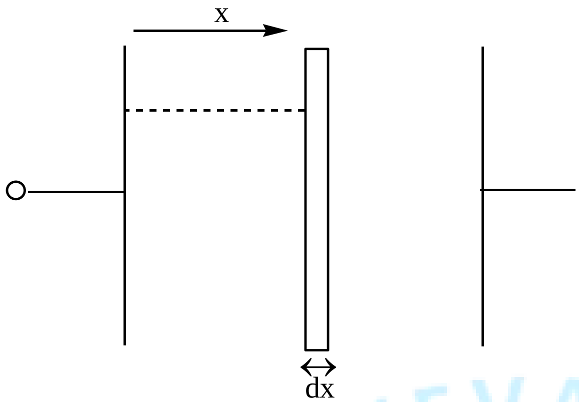
$$= 8.16$$

$$\cong 8$$

Q.16. A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the m^{th} layer is $K_m = K\left(1+\frac{m}{N}\right)$. For a very large $N\left(>10^3\right)$, the capacitance C is $\alpha\left(\frac{K\in_o A}{d\,in 2}\right)$. The value of α will be ____.

[$\in_0$ is the permittivity of free space]

KEY: (1.00 (0.99-1.01)*)



Sol:

$$dc = \frac{K \in_0 A}{dx} \qquad \frac{x}{m} = \frac{D}{N}$$

$$= \frac{K\left(1+\frac{mn}{N}\right) \in_0 A}{dx}$$

$$dc = \frac{K\left(1+\frac{Nx}{DN}\right) \in_0 A}{dx}$$

$$\frac{1}{C_{eq}} = \int_{x=0}^{d} \frac{dx}{K \in_0 A\left(1+\frac{x}{D}\right)}; \qquad = \frac{1}{K \in_0 A} \frac{\ln\left(1+\frac{x}{D}\right)_0^d}{\left(\frac{1}{D}\right)}$$

$$\frac{1}{C_{eq}^0} = \frac{D}{K \in_0 A} \ln(2)$$

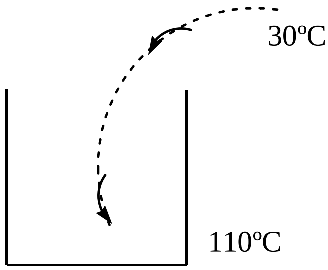
$$C_{eq} = \frac{K \in_0 A}{D \ln 2}$$

K= 1

Q.17. A liquid at 30^0C is poured very slowly into a Calorimeter that is at temperature of 110^0C. The boiling temperature of the liquid is 80^0C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50^0C. The ratio of the Latent heat of the liquid to its specific heat will be _______ ^{0}C.
[Neglect the heat exchange with surrounding]

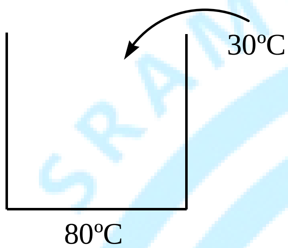
KEY: (270)

Sol:



Heat lost by calorimeter $= m_{cal}S_{cal} \times 30$

Heat gained by liq $= 5 \times S_{liq}.50 + mLy$



Heat lost by calon = $m_{cal}S_{cal} \times 30$

Heat gained by liq = $80 \times S_{liq} \times 20$

$m_{Cal}S_{Cal}30 = S_{eiq}50 + 5L_f$ ___(1)

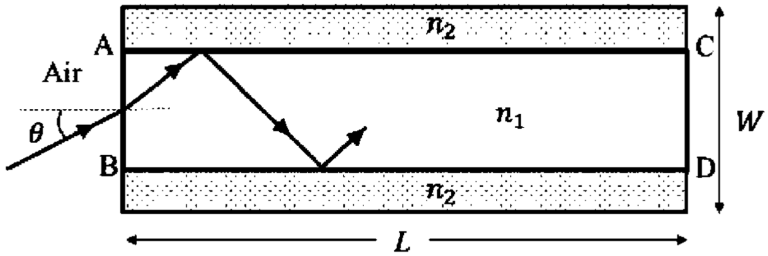
$m_{Cal}S_{Cal} \times 30 = 80S_{eiq}20$___(2)

$1600S_{liq} - 250S_{liq} = 5L_f$

$1350S_{liq} = 5L_f$

$$\frac{L_f}{S_{liq}} = \frac{1350}{5} = 270$$

Q.18. A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If L >> W, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For L = 9.6m, if the incident angle θ is varied, the maximum time taken by a ray of exit the place CD is $t \times 10^{-9}s$, where t is ________.
[Speed of light $c = 3 \times 10^8 m/s$]



KEY: (50(49-51)*)

Sol: $\sin C = \frac{x}{d}$

$$\frac{24}{25} = \frac{x}{d}$$

$$d = \frac{25x}{24}$$

$\rightarrow$ distance travelled by light ray $= \frac{25}{24} \times 9.6$

$= 10M$

Taken by ray to travel

$$\frac{d}{\text{speed in med } n_1}$$

$$\frac{10}{\left(\frac{C}{1.5}\right)} = \frac{1.5 \times 10}{3 \times 10^5}$$

$= 5 \times 10^{-8} S$.

$= 50 \times 10^{-9} S$

JEE (ADVANCED) 2019 PAPER 1

PART-I CHEMISTRY

SECTION 1 (Maximum Marks:12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : -1 In all other cases.

Q.1. The green colour produced in the borax bead test of a chromium (III) salt is due to

(A) $Cr(BO_2)_3$ (B) $Cr_2(B_4O_7)_3$

(C) Cr_2O_3 (D) CrB

KEY: (A)

Sol: $Na_2B_4O_7.10H_2O \xrightarrow{\Delta} Na_2B_4O_7 \xrightarrow{\Delta} Na_2NO_2 + B_2O_3$

$Cr_2O_3 + 3B_2O_3 \rightarrow 2Cr(BO_2)_3$ Chromium meta borate.

Q.2. Calamine, malachite, magnetite and cryolite, respectively, are

(A) $ZnSO_4, CuCO_3, Fe_2O_3, AlF_3$ (B) $ZnSO_4, Cu(OH)_2, Fe_3O_4, Na_3 AlF_6$

(C) $ZnCO_3, CuCO_3 \cdot Cu(OH)_2, Fe_3O_4, Na_3AlF_6$

(D) $ZnCO_3, CuCO_3, Fe_2O_3, Na_3 AlF_6$

KEY: (C)

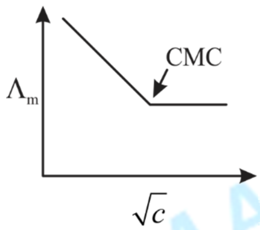
Sol: Calamine $\rightarrow ZnCO_3$

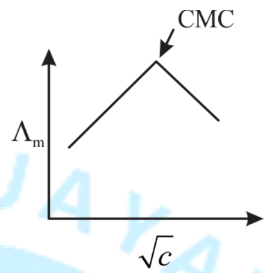
Malachite $\rightarrow CuCO_3.Cu(OH)_2$

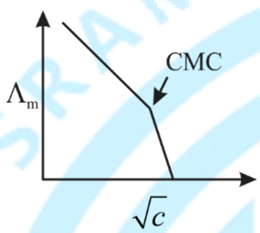
Magnetite $\rightarrow Fe_3O_4$

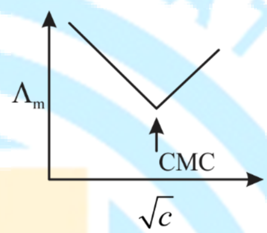
Cryalite $\rightarrow Na_3AlF_6$

Q.3. Molar conductivity (Λ_m) of aqueous solution of sodium stearate, which behaves as a strong electrolyte, is recorded at varying concentrations (c) of sodium stearate. Which one of the following plots provides the correct representation of micelle formation in the solution?
(critical micelle concentration (CMC) is marked with an arrow in the figures)

(A)


(B)


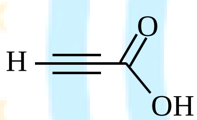
(C)


(D)


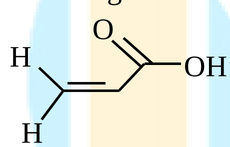
KEY: (C)

Sol: Conceptual

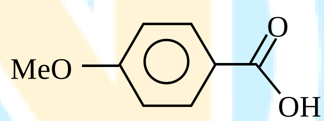
Q.4. The correct order of acid strength of the following carboxylic acids is



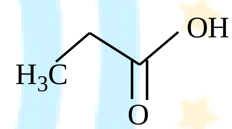
I



II



III

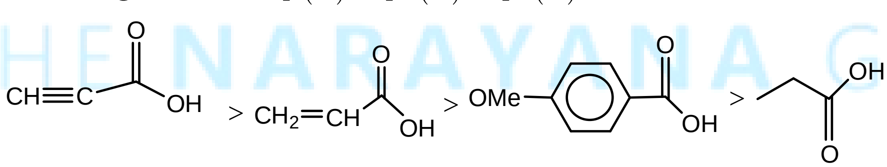


IV

(A) III > II > I > IV
(B) I > II > III > IV
(C) I > III > II > IV
(D) II > I > IV > III

KEY: (B)

Sol: Electronegative order $sp(C) > sp^2(C) > sp^3(C)$.



SECTION 2 (Maximum Marks:32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get -1 mark.

Q.5. A tin chloride Q undergoes the following reactions (not balanced)

$$Q + Cl^- \rightarrow X$$

$$Q + Me_3N \rightarrow Y$$

$$Q + CuCl_2 \rightarrow Z + CuCl$$

X is a monoanion having pyramidal geometry. Both Y and Z are neutral compounds. Choose the correct option(s)

(A) The central atom in X is sp^3 hybridized
(B) There is a coordinate bond in Y
(C) The oxidation state of the central atom in Z is +2
(D) The central atom in Z has one lone pair of electrons

KEY: (A, B)

Sol: $SnCl_2 + Cl^- \rightarrow SnCl_3^- \ (X)$

$SnCl_2 + Me_3N \rightarrow SnCl_2[NMe_3]\,(Y)$

Dative bond between N and Sn

$SnCl_2 + CuCl_2 \rightarrow SnCl_4 + CuCl \ (Z)$

Q.6. Fusion of MnO_2 with KOH in presence of O_2 produces a salt W. Alkaline solution of W upon electrolytic oxidation yields another salt X. The manganese containing ions present in W and X, respectively, are Y and Z. Correct statement(s) is(are)

(A) In aqueous acidic solution, Y undergoes disproportionation reaction to give Z and MnO_2

(B) Both Y and Z are coloured and have tetrahedral shape

(C) Y is diamagnetic in nature while Z is paramagnetic

(D) In both Y and Z, π -bonding occurs between *p*-orbitals of oxygen and *d*-orbitals of manganese

KEY: (A, B, D)

Sol: $MnO_2 + 2KOH + KNO_3 \xrightarrow{\Delta} K_2MnO_4 + H_2O$

$$\underset{(w)}{K_2MnO_4} \longrightarrow \underset{(X)}{KMnO_4}$$

$$\underset{(w)}{K_2MnO_4} \downarrow \underset{(Y)}{MnO_4^{2-}} \qquad \underset{(X)}{KMnO_4} \downarrow \underset{(Z)}{MnO_4^{-}}$$

Q.7. Choose the reaction(s) from the following options, for which the standard enthalpy of reaction is equal to the standard enthalpy of formation.

(A) $2H_2(g)+O_2(g)\rightarrow 2H_2O(l)$

(B) $2C(g)+3H_2(g)\rightarrow C_2H_6(g)$

(C) $\frac{3}{2}O_2(g)\rightarrow O_3(g)$

(D) $\frac{1}{8}S_8(S)+O_2(g)\rightarrow SO_2(g)$

KEY: (C, D)

Sol: Enthalpy change observed when one mole of substance is formed from it constituent elements in their reference states.

Q.8. Which of the following statement(s) is (are) correct regarding the root mean square speed (U_{rms}) and average translational kinetic energy (ε_{av}) of a molecule in a gas at equilibrium ?

(A) U_{rms} is doubled when its temperature is increased four times

(B) ε_{av} is doubled when its temperature is increased four times

(C) ε_{av} at a given temperature does not depend on its molecular mass

(D) U_{rms} is inversely proportional to the square root of its molecular mass

KEY: (A, C, D)

Sol: $E_{av} = \frac{3}{2}RT \qquad V_{rms} \propto \frac{1}{\sqrt{M}}$

$V_{rms} \propto \sqrt{T}$ (absolute temp)

$E_{qv} \propto T$ (absolute temp)

Q.9. Each of the following options contains a set of four molecules. Identify the option(s) where all four molecules possess permanent dipole moment at room temperature.

(A) $BeCl_2, CO_2, BCl_3, CHCl_3$ (B) $NO_2, NH_3, POCl_3, CH_3Cl$

(C) $BF_3, O_3, SF_6, \mathrm{XeF}_6$ (D) $SO_2, C_6H_5Cl, H_2Se, BrF_5$

KEY: (B, D)

Sol: μ value of $BF_3, SF_6, BeCl_2, CO_2, BCl3$
Zero

Q.10. In the decay sequence,

$$^{238}_{92}U \xrightarrow{-x_1} {}^{234}_{90}Th \xrightarrow{-x_2} {}^{234}_{91}Pa \xrightarrow{-x_3} {}^{234}Z \xrightarrow{-x_4} {}^{230}_{90}Th$$

x_1, x_2, x_3 and x_4 are particles/radiation emitted by the respective isotopes. The correct option(s) is(are)

(A) x_1 will deflect towards negatively charged plate

(B) x_2 is β^-

(C) x_3 is γ -ray

(D) Z is an isotope of uranium

KEY: (A, B, D)

Sol: $^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He \quad x_1 = \alpha$

$^{234}_{90}Th \rightarrow {}^{234}_{91}Pa + {}^{0}_{-1}e \quad x_2 = \beta$

$^{234}_{91}Pa \rightarrow {}^{234}_{92}Z + {}^{0}_{-1}e \quad x_3 = \beta$

$^{234}_{92}Z \rightarrow {}^{230}_{90}Th + {}^{4}_{2}He \quad x_4 = \alpha$

Q.11. Which of the following statement(s) is(are) true?

(A) Monosaccharides cannot be hydrolysed to give polyhydroxy aldehydes and ketones

(B) Oxidation of glucose with bromine water gives glutamic acid

(C) Hydrolysis of sucrose gives dextrorotatory glucose and laevorotatory fructose

(D) The two six-membered cyclic hemiacetal forms of D-(+)-glucose are called anomers

KEY: (A, C, D)

Q.12. Choose the correct option(s) for the following set of reactions

(A) $2\mathrm{C(g)} + 3\mathrm{H_2(g)} \longrightarrow \mathrm{C_2H_6(g)}$ (B) $\frac{3}{2}\mathrm{O_2(g)} \longrightarrow \mathrm{O_3(g)}$

(C) $2\mathrm{H_2(g)} + \mathrm{O_2(g)} \longrightarrow 2\mathrm{H_2O}(l)$ (D) $\frac{1}{8}\mathrm{S_8(s)} + \mathrm{O_2(g)} \longrightarrow \mathrm{SO_2(g)}$

KEY: (C, D)

Sol: Enthalpy change observed when one mole of substance is formed from it constituent elements in their reference states.

SECTION 3 (Maximum Marks:18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value of **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is entered;
  Zero Marks : 0 In all other cases.

Q.13. Among $B_2H_6, B_3N_3H_6, N_2O, N_2O_4, H_2S_2O_3$ and $H_2S_2O_8$, the total number of molecules containing covalent bond between two atoms of the same kind is ________.

KEY: (4)

Q.14. At 143 K, the reaction of XeF_4 with O_2F_2 produces a xenon compound Y. The total number of lone pair(s) of electrons present on the whole molecule of Y is ________.

KEY: (19)

Q.15. For the following reaction, the equilibrium constant K_c at 298 K is 1.6×10^{17}.

$$Fe^{2+}(aq)+S^{2-}(aq)\rightleftharpoons FeS(s)$$

When equal volumes of 0.06 M $Fe^{2+}(aq)$ and 0.2 M $S^{2-}(aq)$ solutions are mixed, the equilibrium concentration of $Fe^{2+}(aq)$ is found to be $Y\times10^{-17}M$. The value of Y is __

KEY: (8.93(8.70-9.10)*)

Q.16. On dissolving 0.5g of a non-volatile non-ionic solute to 39 g of benzene, its vapor pressure decreases from 650 mm Hg to 640 mm Hg. The depression of freezing point of benzene (in K) upon addition of the solute is _____.
(Given data: Molar mass and the molal freezing point depression constant of benzene are 78g mol^{-1} and 5.12 K kg mol^{-1}, respectively)

KEY: (1.03(0.97-1.06)*)

Q.17. Consider the kinetic data given in the following table for the reaction $A+B+C\rightarrow$ Product.

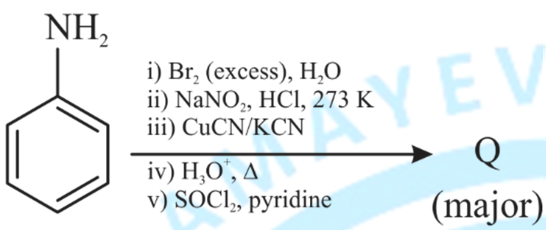
Experiment No.	[A] (mol dm^{-3})	[B] (mol dm^{-3})	[C] (mol dm^{-3})	Rate of reaction (mol $dm^{-3}s^{-1}$)
1	0.2	0.1	0.1	6.0×10^{-5}
2	0.2	0.2	0.1	6.0×10^{-5}
3	0.2	0.1	0.2	1.2×10^{-4}
4	0.3	0.1	0.1	9.0×10^{-5}

The rate of the reaction for $\left[A\right]=0.15\,mol\,dm^{-3},\left[B\right]=0.25\,mol\,dm^{-3}$ and $\left[C\right]=0.15\,mol\,dm^{-3}$ is found to be $Y\times10^{-5}\,mol\,dm^{-3}\,s^{-1}$. The value of Y is _____

KEY: (6.75(6.70-6.80)*)

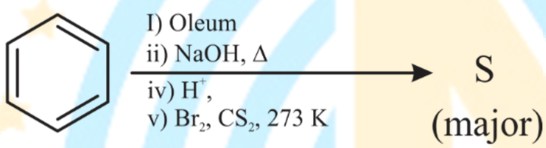
Q.18. Schemes 1 and 2 describe the conversion of P to Q and R to S, respectively. Scheme 3 describes the synthesis of T from Q and S. The total number of Br atoms in a molecule of T is _______.

Scheme 1:



P

Scheme 2:



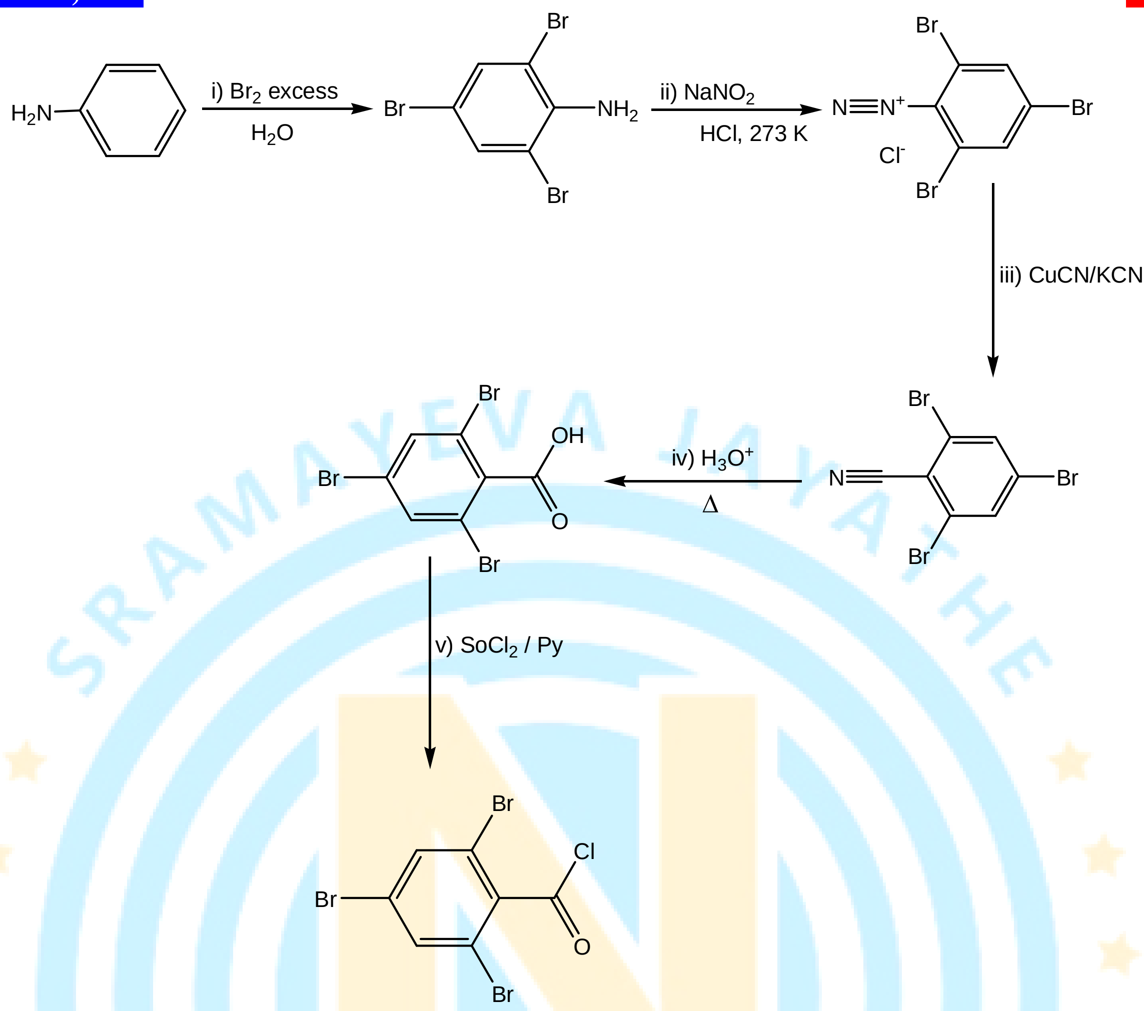
P

Scheme 3:

$S \xrightarrow[\text{ii) Q}]{\text{i) NaOH}} T$ (major)

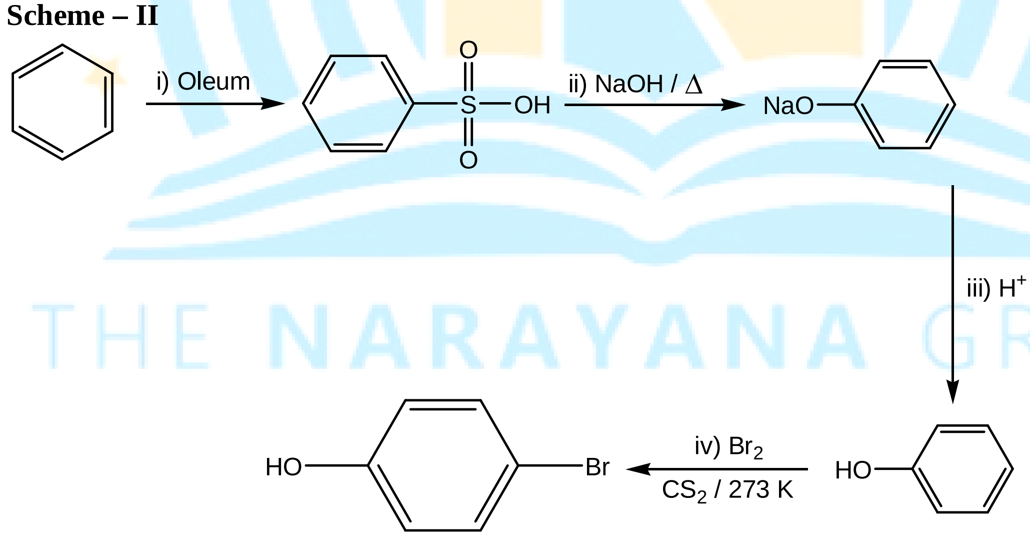
KEY: (4)

Sol: Scheme – I


H2N
i) Br2 excess
H2O
Br
NH2
ii) NaNO2
HCl, 273 K
N≡N+
Cl-
iii) CuCN/KCN
N≡
iv) H3O+
Δ
OH
O
v) SoCl2 / Py
Cl

(Q)

Scheme – II


i) Oleum
S
OH
O
ii) NaOH / Δ
NaO
iii) H+
HO
iv) Br2
CS2 / 273 K
Br

(S)

JEE (ADVANCED) 2019 PAPER 1

PART-I MATHS

SECTION 1 (Maximum Marks:12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct option is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : -1 In all other cases.

Q.1. Let $M=\begin{bmatrix}\sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta\end{bmatrix}=\alpha I+\beta M^{-1}$

Where $\alpha=\alpha(\theta)$ and $\beta=\beta(\theta)$ are real numbers, and I is the 2×2 identity matrix. If α^* is the minimum of the set $\{\alpha(\theta):\theta\in[0,2\pi)\}$ and β^* is the minimum of the set $\{\beta(\theta):\theta\in[0,2\pi)\}$. Then the value of $\alpha^*+\beta^*$ is

1) $-\frac{29}{16}$ 2) $-\frac{37}{16}$ 3) $-\frac{31}{16}$ 4) $-\frac{17}{16}$

KEY: (1)

$$M=\begin{bmatrix}\sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta\end{bmatrix}=\alpha I+\beta M^{-1}$$

Where $\alpha=\alpha(\theta)$ and $\beta=\beta(\theta)$ are real numbers $|M|=(\sin\theta\cos\theta)^4+(1+\sin^2\theta)(1+\cos^2\theta)$

$=2+\sin^2\theta\cos^2\theta+\sin^4\theta\cos^4\theta$

$\because M=\alpha I+\beta M^{-1}$

$$\begin{bmatrix}\sin^4\theta & -(1+\sin^2\theta) \\ 1+\cos^2\theta & \cos^4\theta\end{bmatrix}=\begin{bmatrix}\alpha & 0 \\ 0 & \alpha\end{bmatrix}+\frac{\beta}{|M|}\begin{bmatrix}\cos^4\theta & 1+\sin^2\theta \\ -1-\cos^2\theta & \sin^4\theta\end{bmatrix}$$

Comparing on both side

$$\sin^4\alpha=\alpha+\frac{\beta}{|M|}\cos^4\theta \qquad \text{\_\_\_ (1)}$$

$$-1-\sin^2\theta=\frac{\beta}{|M|}\left(1+\sin^2\theta\right)\Rightarrow\beta=-|M| \_\_\_\_\_ (2)$$

So $\beta=\beta(\theta)=-\left[2+\sin^2\theta\cos^2\theta+\sin^4\theta\cos^4\theta\right]$

$$\beta^*=Minimum\ of\ \beta(\theta)=\frac{-37}{16}\text{ at }\theta=\frac{\pi}{4}+\eta\pi$$

From (1) and (2) $\alpha(\theta)=\sin^4\theta+\cos^4\theta$

$$=1-2\sin^2\theta\cos^2\theta$$

$$=1-\frac{1}{2}\sin^2 2\theta$$

$$\alpha^*=\text{minimum of }\alpha(\theta)=1-\frac{1}{2}=\frac{1}{2}$$

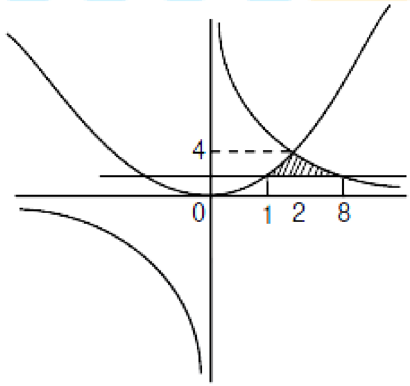
$$\alpha^*+\beta^*=\frac{-37}{16}+\frac{1}{2}=\frac{37+8}{16}=\frac{-29}{16}$$

Q.2. The area of the region $\left\{(x,y):xy\le 8,1\le y\le x^2\right\}$ is

1) $8\log_e 2-\frac{14}{3}$ 2) $8\log_e 2-\frac{7}{3}$ 3) $16\log_e 2-6$ 4) $16\log_e 2-\frac{14}{3}$

KEY: (4)

Sol : The area of the region $\left\{(x,y):xy\le 8,1\le y\le x^2\right\}$



Required area $=\int_1^4\left(\frac{8}{y}-\sqrt{y}\right)dy$

$$=\left[8\ln y-\frac{2^4}{3}y^{3/2}\right]^4$$

$$=\left[8\ln 4-\frac{2}{3}(4)^{3/2}\right]-\left[8(0)-\frac{2}{3}\right]$$

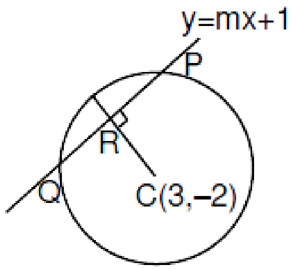
$$=16\ln 2-\frac{16}{3}+\frac{2}{3};\ =16\ln 2-\frac{14}{3}$$

Q.3. A line $y = mx + 1$ intersects the circle $(x-3)^2 + (y+2)^2 = 25$ at the points P and Q. If the midpoint of the line segment PQ has x-coordinate $-\frac{3}{5}$, then which one of the following options is correct ?

1) $4 \le m < 6$ 2) $-3 \le m < -1$ 3) $6 \le m < 8$ 4) $2 \le m < 4$

KEY: (4)

Sol :



$$m \times \frac{\frac{-3m+5}{5}+2}{-\frac{3}{5}-3} = -1; \qquad m \times \frac{-3m+15}{-18} = -1$$

$$3m^2 - 15m + 18 = 0$$
$$m^2 - 5m + 6 = 0$$
$$(m-2)(m-3) = 0 \Rightarrow m = 2,3$$

Q.4. Let S be the set of all complex numbers z satisfying $|z-2+i| \ge \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0-1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|}; z \in S\right\}$, then the principal argument of $\frac{4-z_0-\overline{z_0}}{z_0-\overline{z_0}+2i}$ is

1) $-\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) $\frac{3\pi}{4}$

KEY: (4)

Sol : $|Z-2+i| \ge \sqrt{5} \Rightarrow (x-2)^2 + (y+1)^2 \ge 5$ where Z = x + iy

For $|Z_0 - 1|$ to be minimum, where $Z_0 = x_0 + iy_e$ is at point P on the circle (figure)

$$\text{Arg}\left(\frac{4-Z_0-\overline{Z_0}}{Z_0-\overline{Z_0}+2i}\right)$$

$$= Arg\left(\frac{4-2x_0}{2iy_0+2i}\right)$$

$$= Arg\left(\frac{-i\left(2-x_0\right)}{y_0+1}\right)$$

$$= Arg\left(-i\lambda\right) \text{ where } \lambda = \frac{2-x_0}{y_0+1}$$

$$= -\frac{\pi}{2} \text{ (where } \lambda > 0\text{)}$$

SECTION 2 (Maximum Marks:32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
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  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get -1 mark.

Q.5. Define the collections $\{E_1, E_2, E_3, ...\}$ of ellipses and $\{R_1, R_2, R_3, ...\}$ of rectangles as follows : $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$;

R_1 : rectangle of largest area, with sides parallel to the axes, inscribed in E_1;

E_n : ellipse $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ of largest area inscribed in $R_{n-1}, n > 1$;

R_n : rectangle of largest area, with sides parallel to the axes, inscribed in $E_n, n > 1$.

Then which of the following options is/are correct?

1) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

2) The length of latus rectum of E_9 is $\frac{1}{6}$

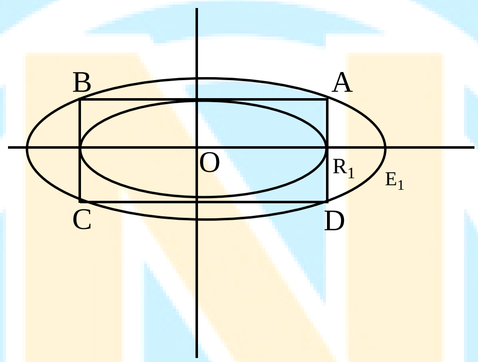
3) The eccentricities of E_{18} and E_{19} are NOT equal

4) $\sum_{n=1}^{N}\left(area\ of\ R_n\right) < 24$, for each positive integer N

KEY: (2, 4)

Sol : $E_1 : \frac{x^2}{9}+\frac{y^2}{4}=1$ (ellipse), $e=\frac{\sqrt{5}}{3}$

R_1 : Max. Rectangle inscribed having sides parallel to axes



For this $A=(3\cos\theta, 2\sin\theta)$, where $\theta=\frac{\pi}{4}$

i.e $A=\left(\frac{3}{\sqrt{2}},\frac{3}{\sqrt{2}}\right)$

$\therefore$ Sides: $3\sqrt{2}, 2\sqrt{2}$ Area of $R_1=\left(3\sqrt{2}\right)\left(2\sqrt{2}\right)$

$\therefore$ If a,b are semi axes of ellipse then $a\sqrt{2}, b\sqrt{2}$ are sides.

For $E_2 : \frac{x^2}{\left(\frac{3}{\sqrt{2}}\right)^2}+\frac{y^2}{\left(\frac{2}{\sqrt{2}}\right)^2}=1$ i.e. semi axes are $\frac{a}{\sqrt{2}},\frac{b}{\sqrt{2}}$.

$\therefore E_9 : \frac{x^2}{\left(\frac{3}{\left(\sqrt{2}\right)^8}\right)^2}+\frac{y^2}{\left(\frac{2}{\left(\sqrt{2}\right)^8}\right)^2} \Rightarrow$ distance from centre to foucs of E_9 = ae $= \frac{3}{\left(\sqrt{2}\right)^8}\times\frac{\sqrt{5}}{3}=\frac{\sqrt{5}}{16}$

$$\text{L.L.R} = \frac{2b^2}{a} = \frac{2\times\left(\frac{2}{\left(\sqrt{2}\right)^8}\right)^2}{\frac{3}{\left(\sqrt{2}\right)^8}} = \frac{2\times\frac{4}{256}}{\frac{3}{16}} = \frac{1}{32}\times\frac{16}{3} = \frac{1}{6}$$

Eccentricities for all ellipses are equal.

Also sum of areas $= R_1 + R_2 + R_3 \ldots\ldots\infty$

$=4\left[\text{Areas sum in } Q_1\right]$

$$=4\left[\frac{3}{\sqrt{2}}\frac{2}{\sqrt{2}}+\frac{3}{\left(\sqrt{2}\right)^2}\frac{2}{\left(\sqrt{2}\right)^2}+\frac{3}{\left(\sqrt{2}\right)^3}\frac{2}{\left(\sqrt{2^3}\right)}+\ldots\ldots\right]$$

$$=(4)(6)\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots\right]$$

$$=24\times\frac{\frac{1}{2}}{1-\frac{1}{2}}=24$$

$$\therefore \sum_{n=1}^{N}\left(\text{area of } R_n\right) < 24$$

Q.6. Let L_1 and L_2 denote the lines $\vec{r} = \hat{i} + \lambda\left(-\hat{i}+2\hat{j}+2\hat{k}\right), \lambda \in R$ and $\vec{r} = \mu\left(2\hat{i}-\hat{j}+2\hat{k}\right), \mu \in R$ respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3?

1) $\vec{r} = \frac{2}{9}\left(2\hat{i}-\hat{j}+2\hat{k}\right)+t\left(2\hat{i}+2\hat{j}-\hat{k}\right), t\in\mathbb{R}$

2) $\vec{r} = t\left(2\hat{i}+2\hat{j}-\hat{k}\right), t\in\mathbb{R}$

3) $\vec{r} = \frac{2}{9}\left(4\hat{i}+\hat{j}+\hat{k}\right)+t\left(2\hat{i}+2\hat{j}-\hat{k}\right), t\in\mathbb{R}$

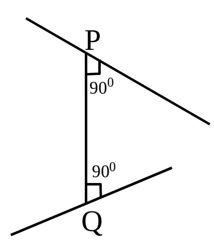
4) $\vec{r} = \frac{1}{3}\left(2\hat{i}+\hat{k}\right)+t\left(2\hat{i}+2\hat{j}-\hat{k}\right), t\in\mathbb{R}$

KEY: (1, 3, 4)

Sol : $L_1 :\ \bar{r} = \bar{i} + (-i+2j+2k)$ .... (1) $\quad \lambda \in R$

$L_2 : \bar{r} = \mu(2i - j + 2k)$ .... (2) $\quad \mu \in R$ given lines

Drs if required line L_3



$(-1,2,2)\times(2,-1,2)$

i.e. 2, 2, -1

Let P on L_1 be $(1-\lambda, 2\lambda, 2\lambda)$

Q on be $(2\mu, \mu, 2\mu)$

$\overline{PQ} \parallel (2,2,-1)$

$\therefore \frac{1-\lambda-2\mu}{2} = \frac{2\lambda+\mu}{2} = \frac{2\lambda-2\mu}{-1}$

Solving $3\lambda+3\mu=1$ .... (1) $\lambda = 1/9$

$6\lambda-3\mu=0$ .... (2) $\mu = 2/9$

$\therefore P = \left(\frac{8}{9}, \frac{2}{9}, \frac{2}{9}\right), Q = \left(\frac{4}{9}, -\frac{2}{9}, \frac{4}{9}\right)$

From choices 1, 3, 4 are correct.

Q.7. Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers *n*, define

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad n \ge 1$$

$b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}, n \ge 2$.

Then which of the following options is/are correct?

1) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

2) $b_n = \alpha^n + \beta^n$ for all $n \ge 1$

3) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

4) $a_1 + a_2 + a_3 + ...a_n = a_{n+2} - 1$ for all $n \ge 1$

KEY: (1, 2, 4)

Sol : Let α, β roots of $x^2 - x - 1 = 0$ $\alpha = \beta$

Clearly $\alpha^2 - \alpha - 1 = 0$ .... (1) $(\alpha^2 = \alpha + 1)$

$\beta^2 - \beta - 1 = 0$ .... (2) $(\beta^2 = \beta + 1)$

Solving $\alpha = \frac{1+\sqrt{5}}{2}, \beta = \frac{1-\sqrt{5}}{2}$

Given $a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \ge 1$

Given $b_1 = 1$, Given $b_n = a_{n-1} + a_{n+1}$

$= \frac{\alpha^{n-1} - \beta^{n-1}}{\alpha - \beta} + \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta}$

$= \frac{\alpha^{n-1}(\alpha^2+1) - \beta^{n-1}(\beta^2+1)}{\alpha - \beta}$

$= \frac{\alpha^{n-1}(\alpha+2) - \beta^{n-1}(\beta+2)}{\alpha - \beta}$

$= \frac{\alpha^{n-1}\left(\frac{5+\sqrt{5}}{2}\right) - \beta^{n-1}\left(\frac{5-\sqrt{5}}{2}\right)}{\sqrt{5}}$

$$= \alpha^{n-1}\left(\frac{\sqrt{5}+1}{2}\right) - \beta^{n-1}\left(\frac{5-\sqrt{5}}{2}\right)$$

$$= \alpha^{n-1}(\alpha) - \beta^{n-1}(-\beta) = \alpha^n + \beta^n$$

$$\therefore b_n = \alpha^n + \beta^n$$

Also $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \sum_{n=1}^{\infty} \frac{\alpha^n + \beta^n}{10^n} = \sum_{n=1}^{\infty}\left(\frac{\alpha}{10}\right)^n + \sum_{n=1}^{\infty}\left(\frac{\beta}{10}\right)^n$

$$= \frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}} = \frac{\alpha}{10-\alpha} + \frac{\beta}{10-\beta} = \frac{10(\alpha+\beta) - 2\alpha\beta}{100 - 10(\alpha+\beta) + \alpha\beta}$$

$$= \frac{10(1) - 2 1 - 1}{100 - 10 + (-1)} = \frac{12}{89}$$

$$\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \sum_{n=1}^{\infty} \frac{\alpha^n - \beta^n}{(\alpha-\beta)10^n} = \frac{1}{\sqrt{5}}\left(\sum_{n=1}^{\infty}\left(\frac{\alpha}{10}\right)^n - \sum_{n=1}^{\infty}\left(\frac{\beta}{10}\right)^n\right)$$

$$= \frac{1}{\sqrt{5}}\left(\frac{\alpha}{10-\alpha} - \frac{\beta}{10-\beta}\right)$$

$$= \frac{1}{\sqrt{5}}\left(\frac{10(\alpha-\beta)}{100 - 10(\alpha+\beta) + \alpha\beta}\right) = \frac{1}{\sqrt{5}}\left(\frac{10 \times \sqrt{5}}{89}\right) = \frac{10}{89}$$

Also $a_1 + a_2 + a_3 + ... + a_n$

$$= \frac{1}{\alpha-\beta}\left[(\alpha-\beta) + (\alpha^2 - \beta^2) + (\alpha^3 - \beta^3) + ... + (\alpha^n - \beta^n)\right]$$

$$= \frac{1}{\alpha-\beta}\left[(\alpha + \alpha^2 + ... + \alpha^n) - (\beta + \beta^2 + ...\beta^n)\right]$$

$$= \frac{1}{\alpha-\beta}\left[(1 + \alpha + \alpha^2 ...... + \alpha^n) - (1 + \beta + \beta^2 ....\beta^n)\right]$$

$$= \frac{1}{\alpha-\beta}\left[\frac{\alpha^{n+1} - 1}{\alpha - 1} - \frac{\beta^{n+1} - 1}{\beta - 1}\right] = \frac{\alpha^{n+2} - \beta^{n+2}}{\alpha - \beta} - 1$$

$$= a_{n+2} - 1$$

Q.8. Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0; \\ x^2 - x + 1, & 0 \le x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \le x < 3; \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \ge 3. \end{cases}$$

Then which of the following options is/are correct?

1) f is onto

2) f is increasing on $(-\infty, 0)$

3) f' has a local maximum at $x = 1$

4) f' is NOT differentiable at $x = 1$

KEY: (1, 3, 4)

Sol : Let $f : R \to R$

$$f(x)=\begin{cases} \dfrac{x^5+5x^4+10x^3+10x^2+3x+1}{x^2-x+1}, & x<0 \\ \dfrac{2}{3}x^3-4x^2+7x-\dfrac{8}{3}, & 0\le x<1 \\ (x-2)\ln(x-2)-x+\dfrac{10}{3}, & 1\le x<3 \\ & x\ge 3 \end{cases}$$

$$f(x)=\begin{cases} (x+1)^5-2x, & x<0 \\ x^2-x+1, & 0\le x\le 1 \\ \dfrac{2}{3}x^3-4x^2+7x-\dfrac{8}{3}, & 1\le x<3 \\ (x-2)\ln(x-2)-x+\dfrac{10}{3}, & x\ge 3 \end{cases}$$

$$f^1(x)=\begin{cases} 5(x+1)^4-2, & x<0 \\ 2x-1, & 0<x<1 \\ 2x^2-8x+7, & 1<x<3 \\ 1+\ln(x-2)-1, & x>3 \end{cases}$$

$$f^{11}(x)=\begin{cases} 20(x+1)^3, & x<0 \\ 2 & 0<x<1 \\ 4x-8 & 1<x<3 \\ \dfrac{1}{x-2} & x>3 \end{cases}$$

$$\left.\begin{array}{r} f(-\infty)=-\infty \\ f(\infty)=\infty \end{array}\right\} \text{Range} = \text{R} \therefore \text{ f is onto}$$

Clearly $f^1\left(-\dfrac{1}{2}\right)=\dfrac{5}{32}-2<0 \therefore f \uparrow atx=-\dfrac{1}{2}$

Also $f^{11}(1-)=2>0$

$f^{11}(1+)<0$

$\therefore f^1(x)$ has local maximum at $x=1$

$$\left.\begin{array}{r} (f^1)^1(1+)=-4 \\ (f^1)(1-)=2 \end{array}\right\} L.D \neq R.D$$

$\therefore f^1$ is not differentiable at x = 1

Q.9. There are bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red 5 green balls, and B_3 contains 5 red and 3 green balls, Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}, \frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a balls is chosen at random from the bag. Then which of the following options is/are correct?

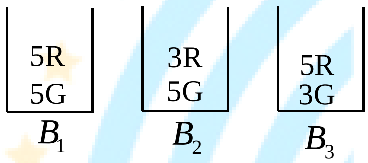
1) Probability that the chosen balls is green equals $\frac{39}{80}$

2) Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{5}{13}$

3) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

4) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

KEY: (1, 3)



Sol :

B_i = Event for choosing bag $B_i, i = 1, 2, 3$.

G = Event for choosing a green ball.

$$P(G) = P(B_1)P\left(\frac{G}{B_1}\right) + P(B_2)P\left(\frac{G}{B_2}\right) + P(B_3)P\left(\frac{G}{B_3}\right)$$

$$= \frac{3}{10} \times \frac{5}{10} + \frac{3}{10} \times \frac{5}{8} + \frac{4}{10} \times \frac{3}{8}$$

$$= \frac{3}{20} + \frac{3}{16} + \frac{3}{20} = \frac{39}{80}$$ Option 1 correct

$$P\left(\frac{B_3}{G}\right) = \frac{P(B_3 \cap G)}{P(G)} = \frac{\frac{4}{10} \times \frac{3}{8}}{\frac{39}{80}} = \frac{4}{13}$$ option 2 incorrect

$$P\left(\frac{G}{B_3}\right) = \frac{3}{8}$$ option 3 is correct

$$P(B_3 \cap G) = P(B_3).P\left(\frac{G}{B_3}\right) = \frac{4}{10} \times \frac{3}{8} = \frac{3}{20}$$ option 4 incorrect

Q.10. Let B denote a curve $y = y(x)$ which is in the first quadrant and let the point $(1,0)$ lie on it. Let the tangent to B at a point P intersect the $y-axis$ at Y_P. If PY_P has length 1 for each point P on B, then which of the following options is/are correct?

1) $y = -\log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

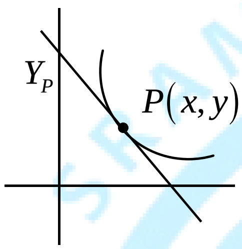
2) $y = \log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$

3) $xy' + \sqrt{1-x^2} = 0$

4) $xy' - \sqrt{1-x^2} = 0$

KEY: (2, 3)

Sol : Equation of tangent at $P(x, y)$



$Xdy - Ydx = xdy - ydx$.

So $Y_p = y - \frac{xdy}{dx}$

$PY_P = \sqrt{x^2 + x^2\left(\frac{dy}{dx}\right)^2} = 1$

$\Rightarrow \frac{dy}{dx} = \pm\frac{\sqrt{1-x^2}}{x}$

So $\int dy = \pm\int\frac{\sqrt{1-x^2}}{x}dx$

$= \pm\left[\int\frac{1-x^2}{x\sqrt{1-x^2}}dx\right]$

$= \pm\left[\int\frac{xdx}{x^2\sqrt{1-x^2}} - \int\frac{xdx}{\sqrt{1-x^2}}\right]$

Let $1-x^2 = t^2 \Rightarrow xdx = -tdt$

So $y = \pm\left[-\int\frac{tdt}{(1-t^2).t} + \int\frac{tdt}{t}\right]$

$= \pm\left[\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + t\right]$

$y = -\ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2} + c$

Or $y = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2} + C$

Putting $(1,0)\, c = 0$ but as function lies in 1st quadrant so $y = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$ &

$$\frac{dy}{dx} = -\frac{\sqrt{1-x^2}}{x}$$

Q.11. In a non-right angled triangle ΔPQR, let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If $p = \sqrt{3}, q = 1$ and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct?

1) Length of $OE = \frac{1}{6}$

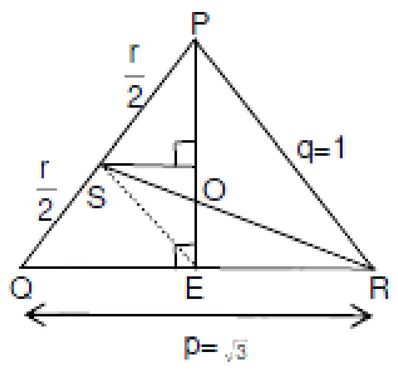
2) Radius of incircle of $\Delta PQR = \frac{\sqrt{3}}{2}\left(2-\sqrt{3}\right)$

3) Area of $\Delta SOE = \frac{\sqrt{3}}{12}$

4) Length of $RS = \frac{\sqrt{7}}{2}$

KEY: (1, 2, 4)

Sol :



$$\frac{\sqrt{3}}{\sin p} = 2 \Rightarrow \sin p = \frac{\sqrt{3}}{2} \text{ so P= 60º, 120º}$$

$$\frac{1}{\sin Q} = 2 \Rightarrow \sin Q = \frac{1}{2} \text{ so Q = 30º, 150º}$$

Clearly P = 60º not possible. So P = 120º, Q = 30º, R = 30º

$$PQ = PR = 1, QE = RE = \frac{\sqrt{3}}{2}$$

O must be centroid so $OE = \frac{1}{3}.PE = \frac{1}{3}\sqrt{1-\frac{3}{4}} = \frac{1}{6}$ option 1 is correct

Area of $\Delta PQR = \frac{1}{2} \times \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{4}$

Semiperimeter $=\left(\sqrt{3}+1+1\right)/2=\dfrac{\sqrt{3}+2}{2}$

So in radius of $\Delta PQR=\dfrac{\frac{\sqrt{3}}{4}}{\frac{\sqrt{3}+2}{2}}=\dfrac{\sqrt{3}\left(2-\sqrt{3}\right)}{2}$ option 2 is correct

$$RS=\sqrt{\frac{PR^2+QR^2-2PS^2}{2}}$$

$=\sqrt{\dfrac{1+3-2\times\frac{1}{4}}{2}}=\dfrac{\sqrt{7}}{2}$ option 4 is correct

Area of $\Delta\ SOE=\dfrac{1}{3}$. Area of ΔSPE

$=\dfrac{1}{3}\times\dfrac{1}{4}5$ area of ΔPQR

$=\dfrac{1}{6}\times\dfrac{1}{2}\times\dfrac{\sqrt{3}}{4}=\dfrac{\sqrt{3}}{48}$ option 3 is incorrect

Q.12. Let $M=\begin{bmatrix}0&1&a\\1&2&3\\3&b&1\end{bmatrix}$ and adj $M=\begin{bmatrix}-1&1&-1\\8&-6&2\\-5&3&-1\end{bmatrix}$ where a and b are real numbers. Which of the following options is/are correct?

1) $a+b=3$

2) $\det\left(adjM^2\right)=81$

3) $\left(adjM\right)^{-1}+adjM^{-1}=-M$

4) If $M\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$, then $\alpha-\beta+\gamma=3$

KEY: (1, 3, 4)

Sol : $AdjM=\begin{pmatrix}2-3b&8&b-6\\ab-1&-3a&3\\3-2a&a&-1\end{pmatrix}^T=\begin{pmatrix}2-3b&ab-1&3-2a\\8&-3a&a\\b-6&3&-1\end{pmatrix}$

Comparing $b-6=-5\Rightarrow b=1$

$3-2a=-1\Rightarrow a=2$

$a+b=3$. Option 1 is correct

$|M|=8+2(1-6)=-2$

$\left|adj\ M^2\right|=\left(M^2\right)^2=|M|^4=16$ option 2 is correct

$\left(adj\ M^{-1}\right)+adj\ M^{-1}=2.\dfrac{M}{|M|}=-M$ option 3 is correct

$$M\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix}=\begin{bmatrix}\beta+2\gamma\\ \alpha+2\beta+3\gamma\\ 3\alpha+\beta+\gamma\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

So $\alpha+\beta+\gamma=1 \Rightarrow 2\alpha=2 \Rightarrow \alpha=1$

$\beta=-1, \gamma=1$

So $\alpha-\beta+\gamma=3$ option 4 is correct

SECTION 3 (Maximum Marks:18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value of **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is entered;
  Zero Marks : 0 In all other cases.

Q.13. Three lines are given by $\vec{r}=\lambda\hat{i}, \lambda\in\mathbb{R}$

$\vec{r}=\mu\left(\hat{i}+\hat{j}\right), \mu\in\mathbb{R}$ and $\vec{r}=v\left(\hat{i}+\hat{j}+\hat{k}\right), v\in\mathbb{R}$.

Let the lines cut the plane $x+y+z=1$ at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals______

KEY: (0.75)

Sol : Let $A(\lambda,0,0)$

$B(\mu,\mu,0)$

$C(\upsilon,\upsilon,\upsilon)$

$\because$ A, B, C are in the plane $x+y+z=1$

$\therefore A(1,0,0)$

$B\left(\frac{1}{2},\frac{1}{2},0\right)$

$C\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

Area of $\Delta ABC=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

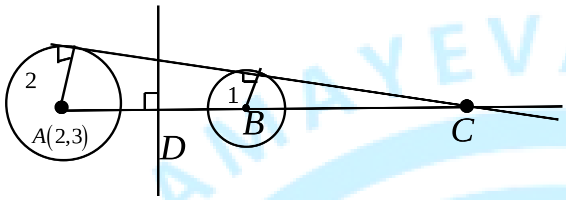
$\Delta=\frac{1}{2}\times\frac{1}{6}\times\sqrt{3}$

$\therefore (6\Delta)^2=3/4=0.75$

Q.14. Let the point B be the reflection of the point $A(2,3)$ with respect to the line $8x-6y-23=0$. Let C_A and C_B be circles of radii 2 and 1 with centers A and B respectively. Let T be a common tangent to the circles C_A and C_B such that both the circles are on the same side of T. If C is the point of intersection of C and the line passing through A and B, then the length of the line segment AC is ______

KEY: (10)

Sol :



$8x-6y-23=0$

$\perp$ distance from A to the line $8x-6y-23=0$ is $\frac{5}{2}$

$\therefore AB=5$, Let $BC=x$

$\therefore \frac{5+x}{x}=\frac{2}{1} \Rightarrow x=5$

$\therefore AC=5+x=10$

Q.15. Let $AP(a;d)$ denote the set of all the terms of an infinite arithmetic progression with first term a and common difference d > 0. If $AP(1;3)\cap AP(2;5)\cap AP(3;7)=AP(a;d)$ then a + d equals________

KEY: (157)

Sol : $AP(1;3)=\{1,4,7,10,..........\}$ say S_1

$AP(2;5)=\{2,7,12,17,..........\}$ say S_2

$AP(3;5)=\{3,10,17,24,..........\}$ say S_3

$S_1\cap S_2=\{7,22,37,52,.......\}$ i.e., $AP(7;15)$

To find the term common to AP (7; 15) and AP (3; 7)

$7+15l=3+7m$

$\therefore m=\frac{4+l}{7}+2l$ where $l,m\in W$

$\Rightarrow l$ can be 3

$\therefore$ m = 7

$\therefore S_1\cap S_2\cap S_3=A.P(52;105)$

$\therefore a+d=157$

Q.16. If $I=\frac{2}{\pi}\int_{-\pi/4}^{\pi/4}\frac{dx}{\left(1+e^{\sin x}\right)\left(2-\cos 2x\right)}$ then $27\,I^2$ equals ____

KEY: (4)

Sol : $I=\frac{2}{\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\frac{dx}{\left(1+e^{\sin x}\right)\left(2-\cos 2x\right)}$

Apply $\int_a^b f\left(x\right)dx=\int_a^b f\left(a+b-x\right)dx$

$$I=\frac{2}{\pi}\int_{\frac{-x}{4}}^{\frac{\pi}{4}}\frac{dx}{\left(1+e^{-\sin x}\right)\left(2-\cos 2x\right)}$$

$$2I=\frac{2}{\pi}\int_{\frac{-x}{4}}^{\frac{\pi}{4}}\frac{dx}{\left(2-\cos 2x\right)}$$

$$I=\frac{1}{\pi}\times 2\int_{b}^{\frac{\pi}{4}}\frac{dx}{2-\left(2\cos^2 x-1\right)}$$

$$=\frac{2}{\pi}\int_0^{\frac{\pi}{4}}\frac{\sec^2 xdx}{3\sec^2 x-2}$$

$$\sec^2 xdx=dt$$

$$=\frac{2}{\pi}\int_0^1\frac{dt}{3\left(t^2+1\right)-2}$$

$$=\frac{2}{\pi}\times\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}t}{1}\right)_0^1$$

$$=\frac{2}{\sqrt{3}\pi}\left(\frac{\pi}{3}-0\right)=\frac{2}{3\sqrt{3}}$$

$$I^2=\frac{4}{27}\Rightarrow 27I^2=4$$

Q.17. Let S be the sample space of all 3×3 matrices with entries from the set $\{0,1\}$. Let the events E_1 and E_2 be given by

$E_1=\{A\in S; Det\,A=0\}$ and $E_2=\{A\in S; sum\ of\ entries\ of\ A\ is\ 7\}$.

If a matrix is chosen at random from S, then the conditional probability $P\left(E_1\mid E_2\right)$ equals_____

KEY: $(\frac{1}{2})$

Sol : $E_1 : |A| = 0$

$$E_2 = a_1 + a_2 + ... + a_a = 7$$

$$P\left(\frac{E_1}{E_2}\right) = \frac{8(E_1 \cap E_2)}{P(E_2)} = \frac{n(E_1 \cap E_2)}{n(E_2)}$$

$$E_2 : 71's, 20's \equiv {}^9C_7 \times {}^2C_2 = 36$$

$$E_2 n E_1 : \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} 3(3)$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} 3(3)$$

$$n(E_2) = 36, n(E_1 \cap E_2) = 18$$

$$P\left(\frac{E_1}{E_2}\right) = \frac{18}{36} = \frac{1}{2}$$

Q.18. Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set $\left\{ \left| a + b\omega + c\omega^2 \right|^2 : a, b, c \; distinct \; non - zero \; integers \right\}$ equals ________

KEY: (3)

Sol : $\left| a + b\omega + c\omega^2 \right|^2 = \left| a + b\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2} \right) + C\left(\frac{-1}{2} - \frac{i\sqrt{3}}{2} \right) \right|^2$

$$= \left| \left(a - \frac{b}{2} - \frac{c}{2} \right) + \frac{i\sqrt{3}}{2}(b - c) \right|^2$$

$$= \left(a - \frac{b}{2} - \frac{c}{2} \right)^2 + \frac{3}{4}(b - c)^2$$

$$= \frac{1}{4}\left[(2a - b - c)^2 + 3(b - c)^2 \right]$$

$$a, b, c \{-1, 0, 1\}$$

Min value : $a = -1, b = 1, c = 0$

$$= \frac{1}{4}\left[(2 - 1)^2 + 3(1)^2 \right] = \frac{12}{4} = 3$$