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HT No. TL0305724	HT No. TL01330751	HT No. TL01327005	HT No. MR16303069	HT No. HR09300789	HT No. TL02305081	Appl. No. 200310166791

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MARCH ATTEMPT

17.03.21_SHIFT - II

MATHEMATICS

MATHEMATICS

1. A triangle ABC in which side AB,BC,CA consist 5,3,6 points respectively, then the number of triangles that can be formed by these points are

(1) 360 (2) 333 (3) 396 (4) 320

Ans. (2)

Sol. Number of triangles = ${}^{14}C_3 - {}^{5}C_3 - {}^{3}C_3 - {}^{6}C_3 = 333$

2. If $(p \wedge q) \otimes (p \oplus q)$ is tautology, then

(1) $\otimes$ is $\rightarrow$ and $\oplus$ is $\vee$ (2) $\otimes$ is $\wedge$ and $\oplus$ is $\wedge$

(3) $\otimes$ is $\vee$ and $\oplus$ is $\vee$ (4) $\otimes$ is $\vee$ and $\oplus$ is $\wedge$

Ans. (1)

Sol.

p	q	$p \wedge r$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

3. The value of $\lim_{n\to\infty} \frac{[r]+[2r]+[3r]+........+[nr]}{n^2}$ is (where [.] represents greatest integer function)

(1) $\frac{r}{2}$ (2) $\frac{r+1}{2}$ (3) 2r (4) 0

Ans. (1)

Sol. $r - 1 < [r] \le r$

$2r - 1 < [2r] \le 2r$

$\vdots$

$nr - 1 < [nr] \le nr$

on adding

$$\underset{h(r)}{\underbrace{\frac{(r+2r+.....+nr)-n}{n^2}}} < \underset{f(r)}{\underbrace{\frac{[r]+[2r]+......+[nr]}{n^2}}} \le \underset{g(r)}{\underbrace{\frac{r+2r+.....+nr}{n^2}}}$$

$$\lim_{n\to\infty} g(r) = \lim_{n\to\infty} \frac{\frac{n(n+1)r}{2}}{n^2} = \frac{r}{2}$$

$$\lim_{n\to\infty} h(r) = \lim_{n\to\infty} \frac{\frac{n(n+1)r}{2} - n}{n^2} = \frac{r}{2}$$

now by sandwich theorem

$$\lim_{n\to\infty} f(r) = \frac{r}{2}$$

4. The tangent at the point P(6,2) to the parabola $y^2 = 4x - 20$ is also tangent to the ellipse $\frac{x^2}{9}+\frac{y^2}{b}=1$. Then the value of 'b' is :

(1) 1 (2) 7 (3) 6 (4) 2

Ans. (2)

Sol. $T : 2y = 2(x + 6) - 20 \Rightarrow y = x - 4$

$\therefore 16 = 9(1) + b \Rightarrow b = 7$

5. If z is a complex number satisfying

$A : |z - 5| \le 1$

$B : \text{Re}((1 - i)z) \ge 1$

$C : \text{Im}(z) \ge 1$, then $n(A \cap B \cap C)$ is

(1) 0 (2) 1 (3) 2 (4) infinite

Ans. (4)

Sol. Let $z = x + iy$

$A : (x - 5)^2 + y^2 \le 1$ ……(i)

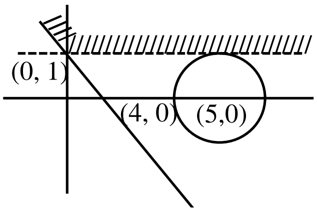
$B : \text{Re}((1 - i)(x + iy)) \ge 1$

$\Rightarrow x + y \ge 1$ ……(ii)

$C : \text{Im}(z) \ge 1$

$\Rightarrow y \ge 1$ ……(iii)

Plotting the regions given by (i), (ii) and (iii)



$\therefore$ n(A ^ B ^ C) is infinite

6. If $f(x) = e^{-x}\sin x$ and $F(x) = \int_0^x f(t)\,dt$ then $\int_0^1 (F'(x)+f(x))\,e^x\,dx$ lies in the interval

(1) $\left(\frac{327}{360},\frac{329}{360}\right)$ (2) $\left(\frac{329}{360},\frac{330}{360}\right)$ (3) $\left(\frac{330}{360},\frac{331}{360}\right)$ (4) $\left(\frac{331}{360},\frac{332}{360}\right)$

Ans. (3)

Sol. $F'(x) = f(x)$ by Leibnitz theorem $\int_0^1 (F'(x)+f(x))\,e^x\,dx = \int_0^1 2f(x)\,e^x\,dx$

$I = \int_0^1 2\sin x\,dx$

$I = 2(1 - \cos 1)$

$$= \left\{1-\left(1-\frac{1^2}{2!}+\frac{1^4}{4!}-\frac{1^6}{6!}+...\right)\right\}$$

$$2\left\{1-\left(1-\frac{1}{2}+\frac{1}{24}\right)\right\} < 2(1-\cos 1) < 2\left\{1-\left(1-\frac{1}{2}+\frac{1}{24}-\frac{1}{720}\right)\right\}$$

$$\frac{330}{360} < 2(1-\cos 1) < \frac{331}{360}$$

$$\frac{330}{360} < I < \frac{331}{360}$$

7. The value of $\sum_{r=0}^{6} {}^6C_r\, {}^6C_{6-r}$ is :

(1) 924 (2) 824 (3) 972 (4) 872

Ans. (1)

Sol. $\sum_{r=0}^{6} {}^6C_r\, {}^6C_{6-r} = {}^{12}C_6 = 924$

8. If $\int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}}dx = \alpha e^{-1} + \beta e^{-\frac{1}{2}} + \gamma$, then $\alpha + \beta + \gamma$ is equal to (where [.] denotes greatest integer function)

(1) 10 (2) 2 (3) 0 (4) 1

Ans. (3)

Sol. $10\int_0^1 \frac{[\sin 2\pi\alpha]}{e^{\{x\}}}dx \Rightarrow 10\left[\int_0^{1/2} 0\ dx + \int_{1/2}^{1} \frac{-1}{e^x}dx\right]$

$$= -10\left[\frac{e^{-x}}{-1}\right]_{1/2}^{1} = 10\left[e^{-1} - e^{-1/2}\right]$$

$$= 10e^{-1} - 10e^{-1/2}$$

$\Rightarrow \alpha = 10$, $\beta = -10, \gamma = 0$

$\Rightarrow \alpha + \beta + \gamma = 0$

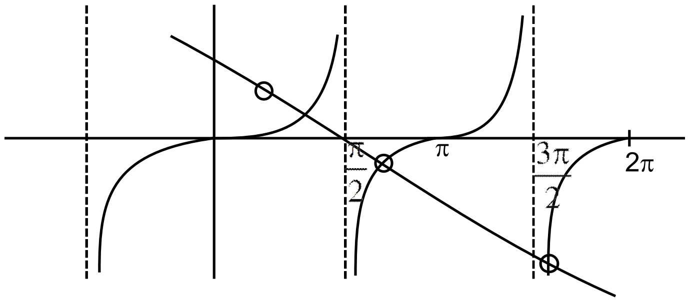
9. Number of solution of the equation $x + 2\tan x = \frac{\pi}{2}$ in $x \in (0, 2\pi)$

(1) 1 (2) 2 (3) 3 (4) 0

Ans. (3)

Sol. $x + 2\tan x = \frac{\pi}{2}$

$\tan x = -\frac{\pi}{2} + \frac{\pi}{4}$



$\therefore$ 3 solutions

10. If $\sin^{-1}\left[x^2 + \frac{1}{3}\right] + \cos^{-1}\left[x^2 - \frac{2}{3}\right] = x^2$ then number of values of x in [–1, 1] is/are (where [.] is GIF)

(1) 0 (2) 1 (3) 2 (4) 3

Ans. (1)

Sol. Case-I: $x \in \left[-1, -\sqrt{\frac{2}{3}}\right)$

$\therefore \sin^{-1}(1) + \cos^{-1}(0) = x^2$

$\Rightarrow x = \pm\sqrt{\pi} \rightarrow$ reject

Case-II: $x \in \left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$

$\sin^{-1}(0) + \cos^{-1}(-1) = x^2$

$\Rightarrow x = \pm\sqrt{\pi} \rightarrow$ Reject

Case-III: $x \in \left[\sqrt{\frac{2}{3}}, 1\right)$

$\sin^{-1}(1) + \cos^{-1}(0) = x^2$

$x^2 = \pi \Rightarrow x = \pm\sqrt{\pi} \rightarrow$ Reject

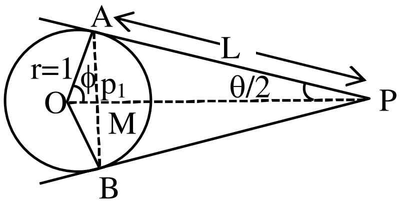
$\therefore$ no solution

11. If a circle $x^2 + y^2 - 4x - 2y + 4 = 0$ from point P, tangents PA & PB are drawn to the given circle and angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, then find $\frac{\text{area (PAB)}}{\text{area(OAB)}}$ where (O is centre of circle)

(1) $\frac{9}{5}$ (2) $\frac{9}{4}$ (3) $\frac{3}{4}$ (4) $\frac{3}{2}$

Ans. (2)

Sol. $\tan\theta = \frac{12}{5} = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} \Rightarrow \tan\frac{\theta}{2} = \frac{2}{3}$



$OA = r = 1, \tan\frac{\theta}{2} = \frac{1}{L}$

$\frac{2}{3} = \frac{1}{L} \Rightarrow L = \frac{3}{2}$

$\phi = \left(\frac{\pi}{2} - \frac{\theta}{2}\right)$

$\tan\phi = \cot\frac{\theta}{2} = \frac{3}{2}$

$\sin\phi = \frac{2}{\sqrt{13}} = \frac{p_1}{1}$

$p_1 = \frac{2}{\sqrt{13}}$

Area of $\Delta OAM = \frac{1}{2} \times \frac{2}{\sqrt{13}} \times \frac{3}{\sqrt{13}} = \frac{3}{13}$

Area of $\Delta OAB = \frac{6}{13}$

Now Area of $\Delta PAB = rL - ar(\Delta OAB) = \frac{3}{2} - \frac{6}{13} = \frac{39-12}{26} = \frac{27}{26}$

Now $\frac{\text{Area }\Delta PAB}{\text{Area }\Delta OAB} = \frac{\frac{27}{26}}{\frac{6}{13}} = \frac{9}{4}$

12. The value of $\lim_{\theta\to 0}\frac{\tan\left(\pi\cos^2\theta\right)}{\sin\left(2\pi\sin^2\theta\right)}$ is equal to

(1) $-\frac{1}{2}$ (2) 0 (3) $\frac{1}{2}$ (4) $\frac{1}{4}$

Ans. (1)

Sol. $\lim_{\theta\to 0}\frac{\tan(\pi-\pi\sin^2\theta)}{\sin(2\pi\sin^2\theta)}=\lim_{\theta\to 0}\frac{-\tan(\pi\sin^2\theta)}{\sin(2\pi\sin^2\theta)}=-\frac{1}{2}$

13. If $f(x)=\begin{cases}\left(2-\sin\frac{1}{x}\right)|x|, & x\neq 0\\ 0 & ,x=0\end{cases}$ then f(x) is

(1) Monotonic in $(-\infty, 0)$ (2) Monotonic in $(0, \infty)$

(3) Monotonic in $(-\infty, 0)\cup(0, \infty)$ (4) Non monotonic in $(-\infty, 0)\cup(0, \infty)$

Ans. (4)

Sol. $f(x)\begin{cases}-\left(2-\sin\frac{1}{x}\right)x\,, & x<0\\ 0 & ,\ x=0\\ \left(2-\sin\frac{1}{x}\right)x & ,\ x>0\end{cases}$

$$f'(x)=\begin{cases}-x\left(-\cos\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)-\left(2-\sin\frac{1}{x}\right), & x<0\\ x\left(-\cos\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)+\left(2-\sin\frac{1}{x}\right), & x>0\end{cases}$$

$$=\begin{cases}-\frac{1}{x}\cos\frac{1}{x}+\sin\frac{1}{x}-2, & x<0\\ \frac{1}{x}\cos\frac{1}{x}-\sin\frac{1}{x}+2, & x>0\end{cases}$$

14. If $\begin{vmatrix}3 & 4\sqrt{2} & x\\ 4 & 5\sqrt{2} & y\\ 5 & k & z\end{vmatrix}=0$ and x, y, z are in A.P. with common difference d, $x\neq 3d$ then value of k^2 is

(1) 36 (2) 72 (3) 6 (4) $6\sqrt{2}$

Ans. (2)

Sol. $\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$

$R_1 \rightarrow R_1 + R_3 - 2R_2$

$\begin{vmatrix} 0 & 4\sqrt{2}+k-10\sqrt{2} & 0 \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$

$\Rightarrow (k - 6\sqrt{2})(4z - 5y) = 0$

$k = 6\sqrt{2}$ or $4z = 5y$

so $k^2 = 72$ $\Rightarrow x = 3d$

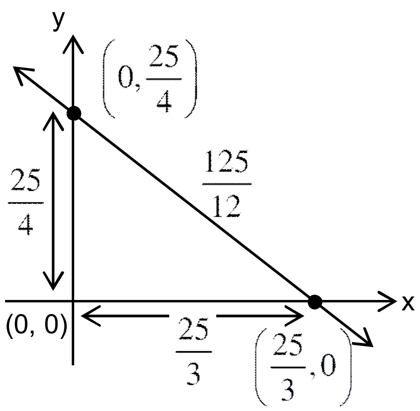
it is not possible

15. Tangent at A(3, 4) of circle $x^2 + y^2 = 25$ meets x and y axis at P and Q if a circle having centre as incentre of ΔOPQ and passing through origin has radius r then r^2 is

(1) $\frac{625}{72}$ (2) $\frac{625}{256}$ (3) $\frac{625}{64}$ (4) $\frac{625}{32}$

Ans. (1)

Sol.



T : 3x + 4y = 25

$$I \equiv \left(\frac{\frac{625}{12}}{\frac{25}{4}+\frac{25}{3}+\frac{125}{12}}, \frac{\frac{625}{12}}{\frac{25}{4}+\frac{25}{3}+\frac{125}{12}} \right)$$

$$\therefore I \equiv \left(\frac{625}{75+100+125}, \frac{625}{75+100+125} \right) \equiv \left(\frac{25}{12}, \frac{25}{12} \right)$$

$$\therefore r^2 = \left(\frac{25}{12}\right)^2 + \left(\frac{25}{12}\right)^2 = \frac{625}{72}$$

16. If curve y(x) satisfied by differential equation $2(x^2 + x^{5/4})\,dy - y\,(x + x^{1/4})\,dx = 2x^{9/4}\,dx$ and passing through $\left(1, \frac{4}{3} - \ell n 2\right)$, then value of y(16) is

(1) $\frac{128}{3} - \frac{16}{3}\ell n 9 + \frac{4}{3}\ell n 2$ (2) $\frac{64}{3} - \frac{16}{3}\ell n 9 + \frac{2}{3}\ell n 2$

(3) $\frac{128}{3} + \frac{16}{3}\ell n 9 - \frac{4}{3}\ell n 2$ (4) $\frac{64}{3} + \frac{16}{3}\ell n 9 - \frac{2}{3}\ell n 2$

Ans. (1)

Sol. $\frac{dy}{dx} - \frac{y}{2x} = \frac{x^{5/4}}{(x + x^{1/4})}$

If $= e^{\int -\frac{1}{2x}dx} = e^{-\frac{1}{2}\ell n x} = \frac{1}{\sqrt{x}}$

Solution is $\frac{y}{\sqrt{x}} = \int \frac{1}{\sqrt{x}} \frac{x^{5/4}}{(x + x^{1/4})} dx$

$\frac{y}{\sqrt{x}} = \int \frac{x^{3/4} + 1 - 1}{x^{1/4}(x^{3/4} + 1)} dx = \int \frac{1}{x^{1/4}} dx - \int \frac{1}{x^{1/4}(x^{3/4} + 1)} dx$

$\frac{y}{\sqrt{x}} = \frac{4x^{3/4}}{3} - \frac{4}{3}\ell n\,(x^{3/4} + 1) + C$ $\{\text{at } x = 1, y = \frac{4}{3} - \ell n\,2\}$

$\frac{4}{3} - \ell n\,2 = \frac{4}{3} - \frac{4}{3}\ell n\,2 + C \Rightarrow \left(\frac{4}{3} - 1\right)\ell n 2 = \frac{1}{3}\ell n 2 = C$

at x = 16, $\frac{y}{4} = \frac{4}{3}.8 - \frac{4}{3}\ell n(9) + \frac{1}{3}\ell n 2$

$y = \frac{128}{3} - \frac{16}{3}\ell n 9 + \frac{4}{3}\ell n 2$

17. If cos x(3sin x + cos x + 3)dy = dx + ysin x (3sin x + cos x + 3)dx then $y\left(\frac{\pi}{3}\right)$ equals

(1) $2\ell n\left(\frac{1+\sqrt{3}}{1+2\sqrt{3}}\right)$ (2) $2\ell n\left(\frac{1+2\sqrt{3}}{1+\sqrt{3}}\right)$ (3) $\ell n\left(\frac{2\sqrt{3}-1}{\sqrt{3}+1}\right)$ (4) $\ell n\left(\frac{\sqrt{3}-1}{2\sqrt{3}+1}\right)$

Ans. (1)

Sol. (cos x. dy – sin x. y . dx)(3sin x + cos x + 3) = dx

$\Rightarrow d(y.\cos x) = \frac{dx}{3\sin x + \cos x + 3}$

$\Rightarrow \int d(y.\cos x) = \int \frac{\sec^2 \frac{x}{2}.dx}{2\tan^2 \frac{x}{2} + 6\tan \frac{x}{2} + 4}$

$$\Rightarrow y\,.\,\cos x = \int \frac{\frac{1}{2}\sec^2\frac{x}{2}.dx}{\tan^2\frac{x}{2}+3\tan\frac{x}{2}+2}$$

$$\Rightarrow y\,.\,\cos x = \ln\left|\frac{\tan\frac{x}{2}+1}{\tan\frac{x}{2}+2}\right|$$

$$y\left(\frac{\pi}{3}\right) = 2\ell n\left(\frac{1+\sqrt{3}}{1+2\sqrt{3}}\right)$$

18. In binary input (having 0 and 1 as inputs) probability of 0 comes in even place is $\frac{1}{2}$ and 0 comes in odd place is $\frac{1}{3}$. Find the probability that 01 is followed by 10.

(1) $\frac{2}{9}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{1}{9}$

Ans. (4)

Sol.

0	e	0	e
1	0	0	1

	odd	even
0	$\frac{1}{3}$	$\frac{1}{2}$
1	$\frac{2}{3}$	$\frac{1}{2}$

e	0	e	0
1	0	0	1

req. probability $= 2 \times \frac{1}{3} \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{9}$

19. If image of point A(2, 3, 1) in the line $\frac{x-1}{2} = \frac{y-4}{1} = \frac{z+3}{-1}$ lies on the plane $\alpha x + \beta y + \gamma z = 24$ also the line $\frac{x-1}{1} = \frac{1-y}{2} = \frac{z-6}{15}$ lies in the plane then $\alpha + \beta + \gamma$ is equal to

Ans. (19)

Sol. Let point on L_1 : $\frac{x-1}{2} = \frac{y-4}{1} = \frac{z-2}{-1}$ is

B $(2\lambda + 1, \lambda + 4, -\lambda - 3)$

Now if B is foot of perpendicular of A in L_1, then AB $\perp$ L_1

$2(2\lambda - 1) + 1(\lambda + 1) - (-\lambda - 4) = 0$

$6\lambda + 3 = 0 \Rightarrow \lambda = -\frac{1}{2}$

Hence B $\left(0, \frac{7}{2}, -\frac{5}{2}\right)$

Now image A′ (–2, 4, –6)

Now equation of plane containing A′(–2, 4, –6) and line $L_2 : \frac{x-1}{1} = \frac{y-1}{-2} = \frac{z-6}{15}$ is

$$\begin{vmatrix} x-1 & y-1 & z-6 \\ 1 & -2 & 15 \\ 3 & -3 & 12 \end{vmatrix} = 0$$

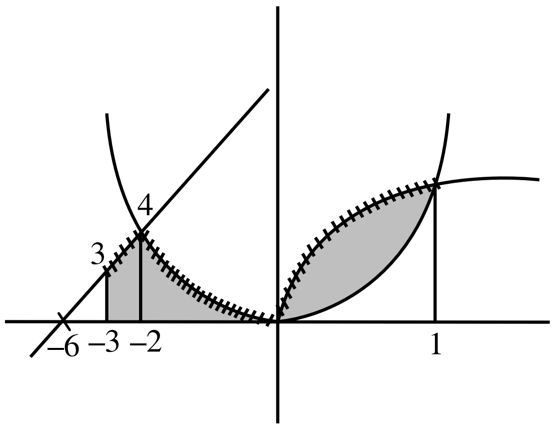
$\Rightarrow 7x + 11y + z = 24$

Hence $\alpha = 7, \beta = 11, y = 1$

20. If area bounded by $f(x) = \begin{cases} \min\{x+6, x^2\} & x \in [-3, 0) \\ \max\{x^2, \sqrt{x}\}, & x \in [0, 1] \end{cases}$ and x-axis is A then find value of 6A

Ans. **41**

Sol.



area is $\int_{-3}^{-2} (x+6)\,dx + \int_{-2}^{0} x^2\,dx + \int_{0}^{1} \sqrt{x}\,dx = A$

$= \frac{7}{2} + \left[\frac{x^3}{3}\right]_{-2}^{0} + \left[\frac{2}{3}x^{3/2}\right]_{0}^{1}$

$= \frac{7}{2} + \frac{8}{3} + \frac{2}{3} = \frac{41}{6}$

So, 6A = 41

21. Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and a + d = 2021 also $B=\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, (where α , $\beta \neq 0$, $\beta \neq 0$), AB = B then ad – bc is equal to

Ans. 2022

Sol. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix}=\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

$$\left.\begin{matrix}\therefore a\alpha+b\beta=\alpha \\ \& c\alpha+d\beta=\beta\end{matrix}\right\} \begin{matrix}\alpha(a-1)=-b\beta \\ c\alpha=\beta(1-d)\end{matrix}$$

$$\frac{a-1}{c}=\frac{b}{d-1}$$

ad – a – d + 1 = bc

$\Rightarrow$ ad – bc = a + d + 1

$\Rightarrow$ ad – bc = 2022

22. For 3n observations of a ungrouped data. Variance is 4 and mean of first 2n observation is 6 and mean of last n observation is 3, if 1 is added to first 2n observations and 1 is subtracted to last n observations then variance of all 3n observation is k then value of 9k is :

Ans. 68

Sol. Let first 2n observations are $x_1, x_2, \ldots\ldots, x_{2n}$ and last n observations are $y_1, y_2, \ldots\ldots y_n$.

Now $\frac{\sum x_i}{2n}=6,\ \frac{\sum y_i}{n}=3 \Rightarrow \sum x_i=12n, \sum y_i=3n$

$$\frac{\sum x_i+\sum y_i}{3n}=\frac{15n}{3n}=5$$

Now $\frac{\sum x_i^2+\sum y_i^2}{3n}-5^2=4$

$$\Rightarrow \sum x_i^2+\sum y_i^2=29\times 3m=87n$$

Now mean is $\frac{\sum(x_i+1)+\sum(y_i-1)}{3n}=\frac{15n+2n-n}{3n}=\frac{16}{3}$

Now variance is $\frac{\sum(x_i+1)^2+\sum(y_i-1)^2}{3n}-\left(\frac{16}{3}\right)^2$

$$=\frac{\sum x_i^2+\sum y_i^2+2\left(\sum x_i-\sum y_i\right)+3n}{3n}-\left(\frac{16}{3}\right)^2$$

$$=\frac{87n+2(9n)+3n}{3n}-\left(\frac{16}{3}\right)^2$$

$$= 29 + 6 + 1 -\left(\frac{16}{3}\right)^2 = \frac{324-256}{9}=\frac{68}{9}=k$$

$\Rightarrow$ 9K = 68

23. Let $f(x) = ax^2 + bx + c \ \forall\ x \in [-1, 1]$, $f(-1) = 2$ and maximum value of $f''(-1)$ is $\frac{1}{2}$ and $f'(-1) = 1$, $f(x) \leq \alpha$ find α_{min}

Ans. 5

Sol. $f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$, $f''(x) = 2a$

given $f''(-1) = \frac{1}{2} \Rightarrow a = \frac{1}{4}$

$f'(-1) = 1 \Rightarrow b - 2a = 1 \Rightarrow b = \frac{3}{2}$

$f(-1) = a - b + c = 2 \quad \Rightarrow c = \frac{13}{4}$

Now $f(x) = \frac{1}{4}(x^2 + 6x + 13)$, $x \in [-1, 1]$

$f'(x) = \frac{1}{4}(2x + 6) = 0 \Rightarrow x = -3 \notin [-1, 1]$

$f(1) = 5$, $f(-1) = 2$

$f(x) \leq 5$

so $\alpha_{minimum} = 5$

24. If coefficient of third, fourth and fifth terms from beginning in the expansion of $\left(x + \frac{a}{x^2}\right)^n$ $(n \in N)$ are in ratio 12 : 8 : 3 then the term independent of x is :

Ans. 4

Sol. $T_{r+1} = {}^nC_r\, x^{n-r}.\left(\frac{a}{x^2}\right)^r$

$= {}^nC_r\, a^r\, x^{n-3r}$

$T_3 = {}^nC_2\, a^2\, x^{n-6}$, $T_4 = {}^nC_3\, a^3\, x^{n-9}$

$T_5 = {}^nC_4\, a^4\, x^{n-12}$

Now $\frac{\text{coefficient of } T_3}{\text{coefficient of } T_4} = \frac{{}^nC_2.a^2}{{}^nC_3a^3} = \frac{3}{a(n-2)} = \frac{3}{2}$

$\Rightarrow a(n-2) = 2 \qquad$ (i)

and $\frac{\text{coefficient } T_4}{\text{coefficient } T_5} = \frac{{}^nC_3.a^3}{{}^nC_4a^4} = \frac{4}{a(n-3)} = \frac{8}{3}$

$\Rightarrow a(n-3) = \frac{3}{2} \qquad$ (ii)

by (i) and (ii) $n = 6$, $a = \frac{1}{2}$

for term independent of 'x'

$n - 3r = 0 \Rightarrow r = \frac{n}{3} \Rightarrow r = \frac{6}{3} = 2$

$T_3 = {}^6C_2\left(\frac{1}{2}\right)^2.x^0 = \frac{15}{4} = 3.75 \approx 4$