

NARAYANA'S SENSATIONAL SUCCESS ACROSS INDIA

7 Students Secured 100 Percentile in All India JEE Main-2020



ADMISSIONS OPEN (2020-21)

OUR REGULAR CLASSROOM PROGRAMME

**One Year Classroom Program
JEE/NEET-2021**
(for students moving from XI to XII)

**Two Year Classroom Program
JEE/NEET-2022**
(for students moving from X to XI)

**Three Year Integrated Classroom Program
JEE/NEET-2023**
(for students moving from IX to X)

**Four Year Integrated Classroom Program
JEE/NEET-2024**
(for students moving from VIII to IX)

**FOUNDATION PROGRAMMES
For NTSE, NSEJS, JSTSE,
Olympiads & School/Board Exams**
(for students moving to
Class VI, VII, VIII, IX & X)

**APEX BATCH
Two years school Integrated
Classroom Program - 2022**
For JEE Main & Advance / NEET (for XI Studying Students)
Course Feature : Complete Coverage of CBSE-Regular Classes-Weekly Test & Regular Analysis-Lab Facility
Motivation & Counseling-Competitive Exam Prep.-Ample time for self study

Online Classes for IIT/NEET/Foundation/Olympiads

- Access Recording of Past Classes on n-Learn App
- Online Parent Teacher Meeting
- Personalized Extra Classes & Live Doubt Solving
- Hybrid/Customized Classroom model
- Video Solution of Weekly/Fortnightly Test
- Printed Study Material will be sent by us
- n-Learn App
- Counselling Motivational sessions
- Affordable Fee
- Doubt Classes / Practice Classes
- Provision to Convert from online to regular classroom programme
- Once Classes resume by just paying nominal fee

Online Test

- Micro & Macro Analysis
- Relative performance (All India Ranking)
- Question wise Analysis
- Unlimited Practice Test
- Grand Test

NARAYANA

Digital Classes
STUDY ONLINE FROM HOME

For Class
7th to 12th +



THE NARAYANA GROUP

JEE-MAIN-2021

MARCH ATTEMPT

18.03.21 SHIFT - I

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MATHEMATICS

MATHEMATICS

- 1.** Let circle

$$C_1 : x^2 + y^2 = 1$$

$$C_2 : x^2 + y^2 - 2y = 1$$

$$C_3 : x^2 + y^2 - 2x = 1$$

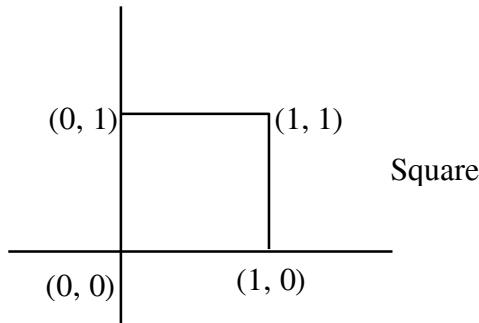
$$C_4 : x^2 + y^2 - 2x - 2y + 1 = 0$$

figure formed by joining centres of C_1, C_2, C_3, C_4 is

- (1) square (2) rectangle (3) rhombus (4) parallelogram

Ans. (1)

Sol. Centres $(0, 0), (1, 0), (0, 1), (1, 1)$



- 2.** The values of a and b for which the function $f(x) = \begin{cases} \frac{1}{|x|}, & |x| \geq 1 \\ ax^2 + b, & |x| < 1 \end{cases}$ is differentiable, are respectively

$$(1) \frac{1}{2}, \frac{3}{2}$$

$$(2) -\frac{1}{2}, \frac{3}{2}$$

$$(3) \frac{1}{2}, -\frac{3}{2}$$

$$(4) 1, -2$$

Ans. (2)

Sol. $f(x)$ is continuous at $x = 1 \Rightarrow 1 = a + b$

$f(x)$ is differentiable at $x = 1 \Rightarrow -1 = 2a$

$$\Rightarrow a = -\frac{1}{2} \therefore b = \frac{3}{2}$$

- 3.** Find the sum of all the four digit numbers formed by using the digits 1,2,2,3.

$$(1) 36664$$

$$(2) 26664$$

$$(3) 24$$

$$(4) 46632$$

Ans. (2)

Sol.1 = $\frac{3!}{2!}$

.....2 = 3!

.....3 = $\frac{3!}{2!}$

sum of digits at unit's place = $3 + 12 + 9 = 24$

sum of all four digits numbers = $24(1111) = 26664$

- 4.** Find the number of points of intersection of the curves :

$$S_1 : x^2 + y^2 - 22x + 10y + 137 = 0 \text{ &}$$

$$S_2 : x^2 + y^2 - 10x - 10y + 41 = 0$$

(1) 1

(2) 2

(3) 0

(4) 3

Ans. (1)

Sol. $S_1 : (x - 11)^2 + (y - 5)^2 = 9 \rightarrow C_1 = (11, 5)$

$$S_2 : (x - 5)^2 + (y - 5)^2 = 9 \rightarrow C_2 = (5, 5)$$

$$r_1 = 3 \text{ & } r_2 = 3$$

$$d(C_1C_2) = \sqrt{(11-5)^2} = 6$$

$$r_1 + r_2 = 6$$

\therefore Circles touch externally

Hence, 1 point of intersection.

- 5.** If $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \dots + a_{40}x^{40}$ then the value of $a_1 + a_2 + \dots + a_{37}$ is :

$$(1) 2^{18}(2^{19} - 19) \quad (2) 2^{19}(2^{20} - 21) \quad (3) 2^{18}(2^{20} - 38) \quad (4) 2^{20}(2^{20} - 21)$$

Ans. (2)

Sol. Put $x = 1, -1$ and subtract

$$4^{20} - 2^{20} = (a_0 + a_1 + \dots + a_{40}) - (a_0 - a_1 + \dots)$$

$$\Rightarrow 4^{20} - 2^{20} = 2(a_1 + a_3 + \dots + a_{39})$$

$$\Rightarrow a_1 + a_3 + \dots + a_{37} = 2^{39} - 2^{19} - a_{39}$$

$$a_{39} = \text{coeff of } x^{39} \text{ in } (1 + x + 2x^2)^{20} = {}^{20}C_1 2^{19}$$

$$\Rightarrow a_1 + a_3 + \dots + a_{37} = 2^{39} - 2^{19} - 20(2^{19}) \\ = 2^{39} - 21(2^{19}) = 2^{19}(2^{20} - 21)$$

- 6.** Find the sum of the series :

$$S = \frac{1}{3^2 - 1} + \frac{1}{5^2 - 1} + \dots + \frac{1}{201^2 - 1}$$

$$(1) \frac{1}{4}$$

$$(2) \frac{25}{101}$$

$$(3) \frac{26}{99}$$

$$(4) \frac{26}{102}$$

Ans. (2)

Sol. $S = \sum_{r=1}^{100} \frac{1}{(2r+1)^2 - 1} = \sum_{r=1}^{100} \frac{1}{(2r+1) \cdot 2(r)}$

$$\therefore S = \frac{1}{4} \sum_{r=1}^{100} \left[\frac{1}{r} - \frac{1}{r+1} \right]$$

$$S = \frac{1}{4} \left(\left(1 - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \dots + \left(\frac{1}{100} - \frac{1}{101} \right) \right)$$

$$\therefore S = \frac{1}{4} \left[\frac{100}{101} \right] = \frac{25}{101}$$

7. A vector \vec{a} has components $3p$ and 1 with respect to a rectangular cartesian system. The system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system, \vec{a} has components $(p+1)$ and $\sqrt{10}$, then find the value of 'p'

(1) $-\frac{5}{4}$

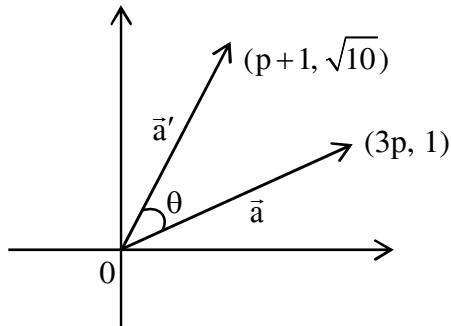
(2) $\frac{4}{5}$

(3) 1

(4) -1

Ans. (4)

Sol.



given $\vec{a} = 3p\hat{i} + \hat{j}$, $\vec{a} = (p+1)\hat{i} + \sqrt{10}\hat{j}$

$|\vec{a}| = |\vec{a}'|$, (No Change in magnitude)

$$\Rightarrow \sqrt{9p^2 + 1} = \sqrt{(p+1)^2 + 10}$$

$$9p^2 + 1 = p^2 + 2p + 1 + 10$$

$$8p^2 - 2p - 10 = 0$$

$$4p^2 - p - 5 = 0$$

$$(4p - 5)(p + 1) = 0$$

$$p = -1, p = \frac{5}{4}$$

8. The value of $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+4}}{\sqrt{4x^2-4x+5}} dx$ is

(1) $\frac{1}{2}\sin\left(\sqrt{4x^2-4x+5}\right) + C$

(2) $\sin\left(\sqrt{4x^2-4x+5}\right) + C$

(3) $\tan\left(\sqrt{4x^2-4x+5}\right) + C$

(4) None of these

Ans. (1)

Sol. Put $\tan^{-1}\left(\frac{2x-1}{2}\right) = \theta$

$$\Rightarrow 2x - 1 = 2\tan\theta$$

$$\Rightarrow I = \int \frac{2\tan\theta \cos(2\sec\theta)}{2\sec\theta} \cdot \sec^2\theta d\theta$$

$$\Rightarrow I = \int \tan\theta \sec\theta \cos(2\sec\theta) d\theta$$

Put $2 \sec \theta = t \Rightarrow 2 \sec \theta \tan \theta d\theta = dt$

$$\Rightarrow I = \frac{1}{2} \int \cos t \, dt = \frac{\sin t}{2} + C = \frac{\sin(2 \sec \theta)}{2} + C$$

$$= \frac{1}{2} \sin(\sqrt{4x^2 - 4x + 5}) + C$$

- 9.** Let $a|z|^2 + \bar{\alpha}z + \bar{z}\alpha + d = 0$ represents the equation of the circle when

- (1) $\alpha^2 - ad > 0$ such that $a \in \mathbb{R}$ (2) $\alpha^2 - ad \geq 0$ such that $a \in \mathbb{R} - \{0\}$
 (3) $\alpha^2 - ad = 0$ for $\forall a \in \mathbb{R}$ (4) $\alpha^2 - ad \leq 0$ for $\forall a \in \mathbb{R}$

Ans. (2)

Sol. $az\bar{z} + \alpha\bar{z} + \bar{\alpha}z + d = 0 \quad \dots \text{(i)}$

$$z\bar{z} + \frac{\alpha}{a}\bar{z} + \frac{\bar{\alpha}}{a}z + \frac{d}{a} = 0 \Rightarrow \text{circle}$$

$$\text{centre} = -\frac{\alpha}{a}, r = \sqrt{|\alpha|^2 - c}$$

$$\Rightarrow \left| \frac{\alpha}{a} \right|^2 - \frac{d}{a} \geq 0 \text{ for equation (i) to represents a circle}$$

$$\Rightarrow |\alpha|^2 - ad \geq 0$$

- 10.** Let the system of equation $\alpha u + \beta v + \gamma w = 0$, $\beta u + \gamma v + \alpha w = 0$, $\gamma u + \alpha v + \beta w = 0$ has non trial

solution and α, β, γ are distinct roots of $x^3 + ax^2 + bx + c = 0$ find value of $\frac{a^2}{b}$

- (1) 0 (2) 1 (3) 2 (4) 3

Ans. (1)

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$$\Rightarrow \alpha^2 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$$

$$\therefore \alpha + \beta + \gamma = 0 \text{ or } \alpha = \beta = \gamma$$

$$\text{Here } \alpha + \beta + \gamma = 0$$

$$\therefore (\alpha, \beta, \gamma \text{ distinct})$$

$$\therefore a = 0$$

- 11.** Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$ and function

$$f(x) + g(x), f(x) - g(x), \frac{f(x)}{g(x)}, \frac{g(x)}{f(x)}, g(x) - f(x)$$

Find the set of value of 'x' which is common to the domain of all the five functions.

- (1) $x \in (0,1)$ (2) $x \in [0,1]$ (3) $x \in [0,1)$ (4) $x \in (0,1]$

Ans. (1)

Sol. $D_f = [0, \infty)$

$$D_g = (-\infty, 1]$$

$$D_{f+g} = D_{f-g} = D_{g-f} = [0, \infty) \cap (-\infty, 1] = [0, 1]$$

for $\frac{f}{g}$ and $\frac{g}{f}$ to be defined, $x \neq 0, 1 \Rightarrow$ common domain = $(0,1)$

- 12.** If $(100)^{\alpha} - (199)^{\beta} = (100)(100) + (99)(101) + (98)(102) + \dots + (1)(199)$. Then find the slope of line joining (α, β) and origin.

- (1) 540 (2) 550 (3) 530 (4) 545

Ans. (2)

$$\text{Sol. RHS} = \sum_{r=0}^{99} (100-r)(100+r)$$

$$= (100)^3 - \frac{99 \times 100 \times 199}{6} = (100)^3 - (1650) 199$$

$$\text{LHS} = (100)^{\alpha} - (199)^{\beta}$$

$$\text{So, } \alpha = 3, \beta = 1650$$

$$\text{Slope} = \tan \theta = \frac{\beta}{\alpha}$$

$$\tan \theta = 550$$

- 13.** Let the straight lines are $y = (mx + 1)$, $3x + 4y = 9$. Find number of integral values of m for which abscissa of point of intersection is integer

- (1) 1 (2) 2 (3) 3 (4) 0

Ans. (2)

$$\text{Sol. } 3x + 4(mx + 1) = 9$$

$$x(3 + 4m) = 5$$

$$x = \frac{5}{(3 + 4m)}$$

$$(3 + 4m) = \pm 1, \pm 5$$

$$4m = -3 \pm 1, -3 \pm 5$$

$$4m = -4, -2, -8, 2$$

$$m = -1, -\frac{1}{2}, -2, \frac{1}{2}$$

Two integral value of m

- 14.** Find the differential equation of the curve $y^2 = 4a(x + a)$ where a is any arbitrary constant.

$$\begin{array}{ll} (1) y(y')^2 + 2xy' - y = 0 & (2) y(y')^2 - 2xy' + y = 0 \\ (3) y(y')^2 + 2xy' + y = 0 & (4) y(y')^2 - 2xy' - y = 0 \end{array}$$

Ans. (1)

$$y^2 = 4a(x + a) \quad \dots\dots \text{(i)}$$

$$2yy' = 4a$$

$$\therefore yy' = 2a$$

$$\therefore \text{by (i)} y^2 = 2yy' \left(x + \frac{yy'}{2} \right)$$

$$y^2 = 2yy'x + (yy')^2$$

$$\Rightarrow y(y')^2 + 2xy' - y = 0$$

(as $y \neq 0$)

- 15.** If $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0, (x \in (0, \pi))$

Then the values of 'x' which satisfy this are :

$$(1) \frac{\pi}{12}, \frac{5\pi}{12} \quad (2) \frac{\pi}{6}, \frac{5\pi}{6} \quad (3) \frac{2\pi}{3}, \frac{\pi}{3} \quad (4) \frac{\pi}{9}, \frac{2\pi}{9}$$

Ans. (1)

Sol. $R_1 \rightarrow R_1 + R_2$

$$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$C_1 \rightarrow C_1 - C_2$

$$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1+\cos^2 x & \cos^2 x \\ 0 & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$$\therefore 2 + 8\sin 2x - 4\sin 2x = 0$$

$$\Rightarrow \sin 2x = \frac{1}{2} \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$$

- 16.** If $2A + B = \begin{bmatrix} 0 & 0 & 3 \\ 10 & 1 & 9 \\ -1 & 4 & 0 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} 0 & -5 & 9 \\ -5 & 3 & 2 \\ -3 & 2 & 0 \end{bmatrix}$

Find the value of $(t_r(A) - t_r(B))$.

$$(1) 1 \quad (2) -2 \quad (3) -1 \quad (4) 2$$

Ans. (4)

Sol. $t_r(2A + B) = 1 \Rightarrow 2t_r(A) + t_r(B) = 1$ (1)

$t_r(A - 2B) = 3 \Rightarrow t_r(A) - 2t_r(B) = 3$ (2)

$\Rightarrow t_r(A) - 2[1 - 2t_r(A)] = 3$

$\Rightarrow t_r(A) = 1, t_r(B) = -1$

$\therefore t_r(A) - t_r(B) = 2$

- 17.** Let $y = (3\sqrt{2})x - 1$ be given line then find equation of straight line passes through A(1, 3) and makes an angle $\tan^{-1}\sqrt{2}$ with given line

(1) $5y + 4\sqrt{2}x = 4\sqrt{2} + 15$

(2) $5y - 4\sqrt{2}x = 4\sqrt{2} + 15$

(3) $4\sqrt{2}x - 5y = 4\sqrt{2} - 15$

(4) None of these

Ans. (1)

Sol. $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$$\sqrt{2} = \left| \frac{m_1 - 3\sqrt{2}}{1 + 3\sqrt{2}m_1} \right|$$

$$\Rightarrow \pm\sqrt{2}(1 + 3\sqrt{2}m_1) = (m_1 - 3\sqrt{2})$$

$$\sqrt{2} + 6m_1 = m_1 - 3\sqrt{2} \quad [\text{for positive sign}]$$

$$m_1 = \frac{-4\sqrt{2}}{5}$$

$$-\sqrt{2} - 6m_1 = m_1 - 3\sqrt{2}$$

$$2\sqrt{2} = 7m_1 \Rightarrow m_1 = \frac{2\sqrt{2}}{7}$$

- 18.** Find number of times, the digit '3' appears while writing integers from 1 to 1000.

Ans. 300

Sol. $\underline{3} \underline{\underline{ }} = (9 \times 9) \times 1$

$$\underline{\underline{3}} \underline{\underline{ }} = (9 \times 9) \times 1$$

$$\underline{\underline{\underline{3}}} \underline{\underline{\underline{ }}} = (9 \times 9) \times 1$$

$$\underline{\underline{\underline{3}}} \underline{\underline{3}} \underline{\underline{ }} = (9) \times 2$$

$$\underline{\underline{3}} \underline{\underline{3}} \underline{\underline{3}} = (9) \times 2$$

$$\underline{\underline{\underline{3}}} \underline{\underline{3}} \underline{\underline{3}} = (1) \times 3$$

$$\text{total way} = 243 + 54 + 3 = 300$$

- 19.** Let P_1 be the plane $x - 2y + 2z - 3 = 0$ and $A(1, 2, 3)$ lie on it. Let there be another plane P_2 which is parallel to P_1 and at unit distance from A. If P_2 is $ax + by + cz + d = 0$ find positive value of $\left(\frac{b-d}{c-a}\right)$

Ans. 4

Sol. Let $P_2 : x - 2y + 2z + \lambda = 0$

$$\left| \frac{1-4+6+\lambda}{\sqrt{1+4+4}} \right| = 1$$

$$|\lambda + 3| = 3$$

$$(\lambda + 3) = \pm 3$$

$$\lambda = -3 \pm 3$$

$$\lambda = 0 \text{ or } -6$$

$$P_2 : x - 2y + 2z + 0 = 0$$

$$x - 2y + 2z - 6 = 0$$

$$\frac{b-d}{c-a} = \frac{(-2) - (0 \text{ or } -6)}{(2-1)} = -2 \text{ or } 4$$

so positive is 4

- 20.** If $f(x^2) + g(4-x) = 4x^3$ and $g(x) + g(4-x) = 0$, then the value of $\int_{-4}^4 f(x^2) dx$

Ans. 512

Sol. $I = 2 \int_0^4 f(x^2) dx \quad \dots \dots \text{(i)}$

$$\Rightarrow I = 2 \int_0^4 f((4-x)^2) dx \quad \dots \dots \text{(ii)}$$

Adding equation (i) & (ii)

$$2I = 2 \int_0^4 [f(x^2) + f(4-x)^2] dx \quad \dots \dots \text{(iii)}$$

$$\text{Now using } f(x^2) + g(4-x) = 4x^3 \quad \dots \dots \text{(iv)}$$

$$x \rightarrow 4-x$$

$$f((4-x)^2) + g(x) = 4(4-x)^3 \quad \dots \dots \text{(v)}$$

Adding equation (iv) & (v)

$$f(x^2) + f(4-x^2) + g(x) + g(4-x) = 4(x^3 + (4-x)^3)$$

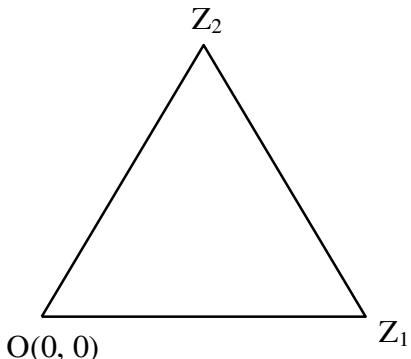
$$\Rightarrow f(x^2) + f(4-x^2) = 4[x^3 + (4-x)^3]$$

$$\text{Now, } I = 4 \int_0^4 (x^3 + (4-x)^3) dx = 512$$

- 21.** If roots of $x^2 + ax + 12 = 0$ and origin form equilateral triangle in complex plane, find $|a|$ equal to

Ans. 6

Sol.



for equilateral triangle

$$Z_1^2 + Z_2^2 + O^2 = Z_1 Z_2 + 0 + 0$$

$$(Z_1 + Z_2)^2 = 3Z_1 Z_2$$

$$\Rightarrow (-a^2) = 3(12)$$

$$\Rightarrow a^2 = 36$$

$$a = -6 \text{ or } 6$$

$$|a| = 6$$

- 22.** Average age of 25 teachers of a school is 40. If a teacher of age 60 year leave the school and a new teacher of 'x' age joined and new mean of their ages is 39 then find the value of 'x'

Ans. 35

$$\text{Sol. } \bar{x} = \frac{\sum_{i=1}^{25} x_i}{25} = 40$$

$$\sum_{i=1}^{25} x_i = 1000$$

when a teacher left school

$$\sum_{i=1}^{24} x_i = 1000 - 60 = 940$$

$$\bar{x}' = \frac{\sum_{i=1}^{24} (x_i) + x}{25}$$

$$39 \times 25 = 940 + x$$

$$x = 35$$

23. If $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3} = \ell$ then

find $6\ell + 1$.

Ans. 4

$$\text{Sol. } \ell = \lim_{x \rightarrow 0} \frac{\left(x + \frac{x^3}{6} + \dots \right) - \left(x - \frac{x^3}{3} \dots \right)}{x^3}$$

$$\therefore \ell = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

$$\therefore 6\ell = 4$$

24. Find number of solutions of equation

$$|\cot x| = \cot x + \frac{1}{\sin x} \text{ in } x \in [0, 2\pi]$$

Ans. 1

$$\text{Sol. Case I : } x \in \left[0, \frac{\pi}{2} \right] \cup \left[\pi, \frac{3\pi}{2} \right]$$

$$\cot x = \cot x + \frac{1}{\sin x} \Rightarrow \text{not possible}$$

$$\text{Case II : } x \in \left[\frac{\pi}{2}, \pi \right] \cup \left[\frac{3\pi}{2}, 2\pi \right]$$

$$-\cot x = \cot x + \frac{1}{\sin x} \Rightarrow \frac{-2\cos x}{\sin x} = \frac{1}{\sin x}$$

$$\Rightarrow \cos x = \frac{-1}{2} \Rightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$= 1$$

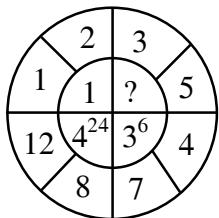
25. If $\int \frac{5x^8 + 7x^6}{(2x^7 + x^2 + 1)^2} dx = f(x) + c$ and if $f(0) = 0$ & $f(1) = \frac{1}{k}$, then find k .

Ans. 4

$$\text{Sol. } \int \frac{5x^8 + 7x^6}{(2x^7 + x^2 + 1)^2} dx = \int \frac{5x^8 + 7x^6}{x^{14} \left(2 + \frac{1}{x^5} + \frac{1}{x^7} \right)^2} dx$$

$$\begin{aligned}
 &= \int \frac{\frac{5}{x^6} + \frac{7}{x^8}}{\left(2 + \frac{1}{x^5} + \frac{1}{x^7}\right)} dx : \text{put } 2 + \frac{1}{x^5} + \frac{1}{x^7} = t \\
 &= \int \frac{-dt}{t^2} = \frac{1}{t} + c \Rightarrow -\left(\frac{5}{x^6} + \frac{7}{x^8}\right) dx = dt \\
 &= f(x) = \frac{1}{2 + \frac{1}{x^5} + \frac{1}{x^7}} = \frac{x^7}{2x^7 + 1 + x^2} \\
 &f(x) = \frac{1}{4} = \frac{1}{k} \Rightarrow k = 4
 \end{aligned}$$

- 26.** Find the missing term in



Ans. 16

Sol. 4^{24} has base 4 ($= 12 - 8$)

3^6 has base 3 ($= 7 - 4$)

(?) will have base 2 ($= 5 - 3$)

Power 24 = 6×4 = (no. of divisor of 12) \times (no. of divisor of 8)

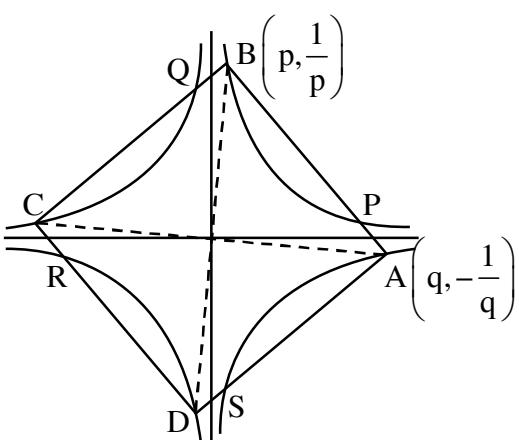
Power 6 = 2×3 = (no. of divisor of 7) \times (no. of divisor of 4)

(?) will have power = (no. of divisor of 3) \times (no. of divisor of 5) = $2 \times 2 = 4$

- 27.** Four points lying on the curve $x^2y^2 = 1$ form a square such that mid-points of sides also lie on the given curve. Find the square of area of the square

Ans. 80

Sol.



OA \perp OB

$$\Rightarrow \left(\frac{1}{p^2} \right) \left(-\frac{1}{q^2} \right) = -1$$

$$\Rightarrow p^2 q^2 = 1$$

$$P \left(\frac{p+q}{2}, \frac{\frac{1}{p} - \frac{1}{q}}{2} \right) \text{ lies}$$

$$\text{on } x^2 y^2 = 1$$

$$\Rightarrow (p+q)^2 \left(\frac{1}{p} - \frac{1}{q} \right)^2 = 16$$

$$\Rightarrow (p+q)^2(p-q)^2 = 16$$

$$\Rightarrow (p^2 - q^2)^2 = 16$$

$$\Rightarrow p^2 - \frac{1}{p^2} = \pm 4$$

$$\Rightarrow p^4 \pm 4p^2 - 1 = 0$$

$$\Rightarrow p^2 = \frac{\pm 4 \pm \sqrt{20}}{2} = \pm 2 \pm \sqrt{5}$$

$$\Rightarrow p^2 = 2 + \sqrt{5} \text{ or } -2 + \sqrt{5}$$

$$OB^2 = p^2 + \frac{1}{p^2} = 2 + \sqrt{5} + \frac{1}{2+\sqrt{5}} \text{ or } -2 + \sqrt{5} + \frac{1}{-2+\sqrt{5}} = 2\sqrt{5}$$

$$\text{Area} = 4 \left(\frac{1}{2} \right) (OA)(OB) = 2(OB)^2 = 4\sqrt{5}$$