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**(Memory Based)
JEE MAIN – 2020
September Session
03-09-2020 (Shift-II)
(MATHEMATICS)**

01. If $\int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{3/2}} dx = \frac{k}{6}$, then $k =$

- 1) $3\sqrt{2} + \pi$ 2) $2\sqrt{3} - \pi$ 3) $2\sqrt{3} + \pi$ 4) $3\sqrt{2} - \pi$

Ans: 2

Sol: Let $x = \sin \theta \Rightarrow dx = \cos \theta d\theta$

Given integral becomes $\frac{k}{6} = \int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{3/2}} dx$

$$\Rightarrow \frac{k}{6} = \int \frac{\sin^2 \theta}{(1-\sin^2 \theta)^{3/2}} \cdot \cos \theta d\theta \Rightarrow \frac{k}{6} = \int_0^{\frac{\pi}{6}} \frac{\sin^2 \theta}{\cos^3 \theta} \cos \theta d\theta$$

$$\Rightarrow \frac{k}{6} = \int_0^{\frac{\pi}{6}} \tan^2 \theta d\theta = \int_0^{\frac{\pi}{6}} (\sec^2 \theta - 1) d\theta \Rightarrow \frac{k}{6} = (\tan \theta - \theta)_0^{\pi/6} = \left(\frac{1}{\sqrt{3}} - \frac{\pi}{6} \right) = \frac{2\sqrt{3} - \pi}{6} \Rightarrow k = 2\sqrt{3} - \pi$$

02. Let $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ are ellipse and hyperbola having eccentricities e_1, e_2 respectively such that $e_1 e_2 = 1$. If distance between foci of ellipse is α and that of hyperbola is β then (α, β) is equal to

- 1) (4,5) 2) (8,10) 3) (10,7) 4) (4,10)

Ans: 2

Sol: $e_1 = \sqrt{1 - \frac{b^2}{25}}$; $e_2 = \sqrt{1 + \frac{b^2}{16}}$; $e_1 e_2 = 1$

$$\Rightarrow (e_1 e_2)^2 = 1 \Rightarrow \left(1 - \frac{b^2}{25}\right) \left(1 + \frac{b^2}{16}\right) = 1 \Rightarrow 1 + \frac{b^2}{16} - \frac{b^2}{25} - \frac{b^4}{25 \times 16} = 1$$

$$\Rightarrow \frac{9}{16 \times 25} b^2 - \frac{b^4}{25 \times 16} - 0 \Rightarrow b^2 = 9$$

$$\alpha = 2(5)(e_1) = 8 \quad ; \quad \beta = 2(4)(e_2) = 10$$

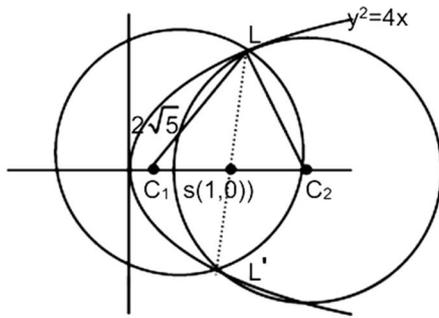
$$(\alpha, \beta) = (8, 10)$$

03. Two equal circles of radius $2\sqrt{5}$ pass through extremities of latusrectum of $y^2 = 4x$, then the distance between centers of circles is

- 1) 4 2) 8 3) 2 4) 6

Ans: 2

Sol:



Given parabola is $y^2 = 4x$, Focus is $S=(1,0)$ and $LR=4$

$$\text{From triangle } C_1LS, (C_1L)^2 = (C_1S)^2 + (LS)^2 \Rightarrow 20 = (C_1S)^2 + 4$$

$$\Rightarrow C_1C_2 = 2C_1S = 2\sqrt{20-4} = 8$$

04. If $\int \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right) dx = A(x) \tan^{-1}\sqrt{x} + B(x) + C$, then $A(x)$ and $B(x)$ will be

- 1) $1+x, \sqrt{x}$ 2) $1-x, -\sqrt{x}$ 3) $1+x, -\sqrt{x}$ 4) $1-x, \sqrt{x}$

Ans: 3

$$\begin{aligned} \text{Sol: } I &= \int \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right) dx, \left(\text{Let } \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right) = A \Rightarrow \sin A = \frac{\sqrt{x}}{\sqrt{1+x}} \Rightarrow \tan A = \sqrt{x} \Rightarrow A = \tan^{-1}(\sqrt{x})\right) \\ &= \int \tan^{-1}(\sqrt{x}) dx = x \tan^{-1}\sqrt{x} - \int \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} \cdot x dx + C = x \tan^{-1}\sqrt{x} - \frac{1}{2} \int \frac{t \cdot 2t \cdot dt}{1+t^2} + C \quad (\text{Put } x=t^2) \\ &= x \tan^{-1}\sqrt{x} - \int \frac{t^2}{1+t^2} dt + C = x \tan^{-1}\sqrt{x} - t + \tan^{-1}(t) + C = x \tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C \\ &= (x+1) \tan^{-1}\sqrt{x} - \sqrt{x} + C \Rightarrow (Ax) = x+1 \Rightarrow B(x) = -\sqrt{x} \end{aligned}$$

(OR)

Put $x = \tan^2 \theta$ then apply integration by parts.

05. The coefficient of term independent of x in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ is λ , then 18λ is

- 1) 9 2) 7 3) 6 4) 4

Ans:(2)

$$\text{Sol: } T_{r+1} = {}^9C_r \left(\frac{3x^2}{2}\right)^{9-r} \left(-\frac{1}{3x}\right)^r = {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r x^{18-3r}$$

For the term independent of x , put $18 - 3r = 0 \Rightarrow r = 6$

$$\text{Now } T_7 = {}^9C_6 \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6 = {}^9C_6 \left(\frac{1}{6}\right)^3 = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} \left(\frac{1}{6}\right)^3 = \left(\frac{7}{18}\right)$$

06. If $|Z_1 - 1| = \operatorname{Re}(Z_1)$, $|Z_2 - 1| = \operatorname{Re}(Z_2)$ and $\operatorname{Arg}(Z_1 - Z_2) = \frac{\pi}{3}$, then $\operatorname{Im}(Z_1 + Z_2) =$

- 1) $\frac{1}{\sqrt{3}}$ 2) $\frac{2}{\sqrt{3}}$ 3) $\frac{\sqrt{3}}{2}$ 4) $\sqrt{3}$

Ans: 2

Sol:

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$|Z_1 - 1| = \operatorname{Re}(Z_1) \Rightarrow (x_1 - 1)^2 + y_1^2 = x_1^2 \Rightarrow y_1^2 - 2x_1 + 1 = 0 \quad \dots\dots\dots(1)$$

$$|z_2 - 1| = \operatorname{Re}(z_2) \Rightarrow (x_2 - 1)^2 + y_2^2 = x_2^2 \Rightarrow y_2^2 - 2x_2 + 1 = 0 \quad \dots\dots\dots(2)$$

$$(1) - (2) \Rightarrow y_1^2 - y_2^2 - 2(x_1 - x_2) = 0 \Rightarrow (y_1 - y_2)(y_1 + y_2) = 2(x_1 - x_2) \Rightarrow y_1 + y_2 = 2 \left(\frac{x_1 - x_2}{y_1 - y_2} \right) \dots\dots(3)$$

$$\operatorname{Arg}(z_1 - z_2) = \frac{\pi}{3} \Rightarrow \tan^{-1} \left(\frac{y_1 - y_2}{x_1 - x_2} \right) = \frac{\pi}{3} \Rightarrow \frac{y_1 - y_2}{x_1 - x_2} = \sqrt{3} \quad \dots\dots\dots(4)$$

$$\therefore y_1 + y_2 = \frac{2}{\sqrt{3}} \Rightarrow \operatorname{Im}(z_1 + z_2) = \frac{2}{\sqrt{3}}$$

07. The probability that a randomly selected 5 digit number contain exactly two distinct digits is

- 1) $\frac{135}{10^4}$ 2) $\frac{125}{10^4}$ 3) $\frac{144}{10^4}$ 4) $\frac{127}{10^4}$

Ans: 1

Sol: Total number of ways = $9(10^4)$

Favourable number of ways = Number of 5 digit numbers excluding '0' having exactly 2 distinct digits + Number of 5 digit numbers including '0' having exactly two distinct digits

$$= {}^9C_2(2^5 - 2) + {}^9C_1(2^4 - 1) = 36(30) + 9(15) = 1080 + 135$$

$$\text{Probability} = \frac{36 \times 30 + 9 \times 15}{9 \times 10^4} = \frac{4 \times 30 + 15}{10^4} = \frac{135}{10^4}$$

08. Let $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ be a quadratic equation, then set of values of λ if exactly one root of the equation lies in the interval $(0, 1)$ is

- 1) $(2, 3)$ 2) $(1, 3)$ 3) $[1, 2)$ 4) $(1, 3]$

Ans: 4

Sol: $f(0)f(1) \leq 0 \Rightarrow 2(\lambda^2 + 1 - 4\lambda + 2) \leq 0 \Rightarrow 2(\lambda^2 - 4\lambda + 3) \leq 0 \Rightarrow (\lambda - 1)(\lambda - 3) \leq 0 \Rightarrow \lambda \in [1, 3]$

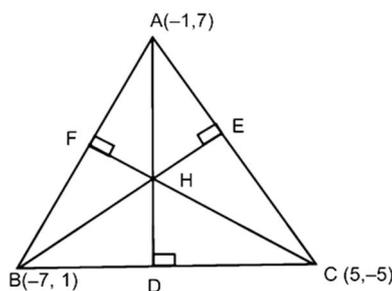
But at $\lambda = 1$, both roots are 1. So $\lambda \neq 1 \therefore \lambda \in (1, 3]$

09. The orthocenter of ΔABC whose vertices are $A(-1, 7)B(-7, 1)C(5, -5)$ is

- 1) $(-3, 3)$ 2) $(3, -3)$ 3) $(3, 3)$ 4) $(-3, -3)$

Ans: 1

Sol:



$$m_{BC} = \frac{6}{-12} = -\frac{1}{2} \therefore \text{Equation of AD is } y-7 = 2(x+1) \Rightarrow y = 2x+9 \dots\dots\dots(1)$$

$$m_{AC} = \frac{12}{-6} = -2, \text{ Equation of BE is } y-1 = \frac{1}{2}(x+7) \Rightarrow y = \frac{x}{2} + \frac{9}{2} \dots\dots\dots(2)$$

$$\text{By (1) and (2) ; } 2x+9 = \frac{x+9}{2} \Rightarrow 4x+18 = x+9 \Rightarrow 3x = -9 \Rightarrow x = -3$$

From (1), $y = 3$. Therefore orthocenter of triangle ABC is $(-3,3)$

10. If relation $R_1 = \{(a,b) : a,b \in R, a^2 + b^2 \in Q\}$ and $R_2 = \{(a,b) : a,b \in R, a^2 + b^2 \notin Q\}$ then

- 1) R_1 is transitive, R_2 is not transitive
- 2) R_1 is not transitive R_2 is not transitive
- 3) R_1 is transitive R_2 is transitive
- 4) R_1 is not transitive R_2 is transitive

Ans. (2)

Sol. For R_1 , let $a = 1 + \sqrt{2}, b = 1 - \sqrt{2}, c = 8^{1/4}$

$$aR_1b \Rightarrow a^2 + b^2 = 6 \in Q$$

$$aR_1c \Rightarrow b^2 + c^2 = (1 - \sqrt{2})^2 + (8^{1/4})^2 = 3 \in Q$$

$$aR_1c \Rightarrow a^2 + c^2 = (1 + \sqrt{2})^2 + (8^{1/4})^2 = 3 + 4\sqrt{2} \notin Q$$

R_1 is not transitive.

For R_2 , let $a = 1 + \sqrt{2}, b = \sqrt{2}, c = 1 - \sqrt{2}$

$$aR_2b \Rightarrow a^2 + b^2 = (1 + \sqrt{2})^2 + (\sqrt{2})^2 = 5 + 2\sqrt{2} \notin Q$$

$$bR_2b \Rightarrow b^2 + c^2 = (\sqrt{2})^2 + (1 - \sqrt{2})^2 = 5 - 2\sqrt{2} \notin Q$$

$$aR_2c \Rightarrow a^2 + c^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 6 \in Q$$

$\therefore R_2$ is not transitive.

11. If the sum of first n terms of series $20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \dots$ is 488 and n^{th} term is negative then the

n^{th} term is

1) -4

2) 4

3) 1

4) 6

Ans. (4)

$$\text{Sol. } 488 = \frac{n}{2} \left[2 \left(\frac{100}{5} \right) + (n-1) \left(-\frac{2}{5} \right) \right]$$

$$488 = \frac{n}{5} (101 - n) \Rightarrow n^2 - 101n + 2440 = 0$$

$$\Rightarrow n = 61 \quad (\text{or}) \quad 40$$

$$\text{For } n = 40, \quad T_n > 0$$

Therefore $n = 61$

$$n^{\text{th}} \text{ term} = 20 + 60 \left(\frac{-2}{5} \right) = -4$$

12. Surface area of cube is increasing at rate of $3.6 \text{ cm}^2/\text{s}$, then the rate at which its volume is increasing at the moment, when length of edge is 10 cm., is

1) 9

2) 10

3) 18

4) 20

Ans. (1)

$$\text{Sol. } S = 6a^2 \Rightarrow \frac{ds}{dt} = 12a \cdot \frac{da}{dt} = 3.6 \Rightarrow 12(10) \frac{da}{dt} = 3.6 \Rightarrow \frac{da}{dt} = 0.03$$

$$V = a^3 \Rightarrow \frac{dv}{dt} = 3a^2 \cdot \frac{da}{dt} = 3(10)^2 \cdot \left(\frac{3}{100} \right) = 9$$

13. Which of the following point lies on plane containing lines

$$\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j} + \hat{k}) \quad \text{and} \quad \vec{r} = -\hat{j} + \mu(-\hat{i} - 2\hat{j} + \hat{k}) \quad \text{is}$$

1) (1,3,6)

2) (1,-3,6)

3) (-2,1,2)

4) (1,3,1)

Ans. (2)

$$\text{Sol. Normal of plane is } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & -2 & 1 \end{vmatrix} \quad \text{i.e., } \vec{n} = 3\hat{i} - 2\hat{j} - \hat{k}$$

D.R.s normal to the plane are 3, -2, -1

$$\text{Equation of the Plane is } 3(x-1) - 2(y-0) - 1(z-0) = 0 \Rightarrow 3x - 2y - z - 3 = 0$$

$$14. \lim_{x \rightarrow a} \frac{(a^2 + 2x^2)^{1/3} - (3x^2)^{1/3}}{(3a^2 + x^2)^{1/3} - (4x^2)^{1/3}} =$$

1) $\left(\frac{4}{3}\right)^{2/3}$ 2) $\frac{1}{3} \left(\frac{3}{4}\right)^{2/3}$ 3) $\frac{1}{3} \left(\frac{2}{3}\right)^{2/3}$ 4) $\frac{1}{3} \left(\frac{4}{3}\right)^{2/3}$

Ans. (4)

Sol. $\lim_{x \rightarrow a} \frac{\frac{1}{3}(a^2 + 2x^2)^{-2/3} \cdot 4x - \frac{1}{3} \cdot (3x^2)^{-2/3} \cdot 6x}{\frac{1}{3}(3a^2 + x^2)^{-2/3} \cdot 2x - \frac{1}{3}(4x^2)^{-2/3} \cdot 8x} = \frac{(3a^2)^{-2/3} \times 2}{(4a^2)^{-2/3} \times 6} = \frac{1}{3} \cdot \frac{4^{2/3}}{3^{2/3}}$

15. If $x^3 dy + xy dx = 2y dx + x^2 dy$ & $y(2) = e$, then $y(4)$ is equal to

- 1) $\frac{1}{2} + \sqrt{e}$ 2) $\frac{1}{2} \sqrt{e}$ 3) \sqrt{e} 4) $\frac{3}{2} \sqrt{e}$

Ans. (4)

Sol. $\Rightarrow x^3 dy + xy dx = 2y dx + x^2 dy \Rightarrow \frac{dy}{y} = \frac{2-x}{x^2(x-1)} dx$

$\Rightarrow \int \frac{dy}{y} = \int \frac{2-x}{x^2(x-1)} dx \dots\dots\dots(i)$

Let $\frac{2-x}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$

$\Rightarrow 2-x = A(x-1) + B(x-1) + Cx^2 \Rightarrow C = 1, B = -2 \text{ and } A = -1$

$\Rightarrow \int \frac{dy}{y} = \int \left\{ \frac{-1}{x} - \frac{2}{x^2} + \frac{1}{x-1} \right\} dx \Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln|x-1| + C$

$\therefore y(2) = e$

$\Rightarrow 1 = -\ln 2 + 1 + 0 + C \Rightarrow C = \ln 2$

$\Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln|x-1| + \ln 2$

at $x = 4$

$\Rightarrow \ln y(4) = -\ln 4 + \frac{1}{2} + \ln 3 + \ln 2$

$\Rightarrow \ln y(4) = \ln\left(\frac{3}{2}\right) + \frac{1}{2} = \ln\left(\frac{3}{2} e^{1/2}\right) \Rightarrow y(4) = \frac{3}{2} e^{1/2}$

16. If $\frac{a}{\cos \theta} = \frac{b}{\cos\left(\theta + \frac{2\pi}{3}\right)} = \frac{c}{\cos\left(\theta + \frac{4\pi}{3}\right)}$, then the angle between vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$

, where $\theta = \frac{2\pi}{9}$ and $a^2 + b^2 + c^2 = 1$, is

- 1) $\frac{\pi}{3}$ 2) $\frac{\pi}{6}$ 3) $\frac{2\pi}{3}$ 4) $\frac{5\pi}{6}$

Ans: (3)

Sol: $\frac{a}{\cos \theta} = \frac{b}{\cos\left(\theta + \frac{2\pi}{3}\right)} = \frac{c}{\cos\left(\theta + \frac{4\pi}{3}\right)} = \frac{a+b+c}{\cos \theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)} = \frac{a+b+c}{0}$

$$\Rightarrow a+b+c=0 \Rightarrow a^2+b^2+c^2+2(ab+bc+ca)=0 \Rightarrow ab+bc+ca=-\frac{1}{2}$$

Now let angle between given vectors is ϕ

$$\therefore \cos \phi = \frac{(\hat{a}\hat{i} + \hat{b}\hat{j} + \hat{c}\hat{k})(\hat{b}\hat{i} + \hat{c}\hat{j} + \hat{a}\hat{k})}{a^2+b^2+c^2} \Rightarrow \cos \phi = \frac{ab+bc+ca}{1} = \frac{-1}{2} \Rightarrow \phi = \frac{2\pi}{3}$$

17. If $(p \wedge q) \rightarrow (\sim q \vee r)$ has truth value false then the truth values of p,q,r respectively are

- 1) T,T,F 2) T,F,T 3) F,FT 4) T,T,T

Ans: 1

Sol: $(p \wedge q)$ should be TRUE and $(\sim q \vee r)$ should be FALSE.

By observing truth table we can conclude that $p = T, q = T, r = F$

18. If m A.M's and 3 G.M's are inserted between 3 and 243 such that 2nd GM = 4thAM, then m is equal to

Ans. 39.00

Sol. 3, $A_1, A_2, A_3, \dots, A_m, 243$ are in A.P

$$\text{Common Difference is } d = \frac{243-3}{m+1} = \frac{240}{m+1}$$

3, $G_1, G_2, G_3, 243$ are in G.P

$$\text{Common Ratio is } r = \left(\frac{243}{3}\right)^{\frac{0}{3+1}} = (81)^{1/4} = 3$$

$$\text{Given } G_2 = A_4 \Rightarrow 3(3)^2 = 3 + 4\left(\frac{240}{m+1}\right) \Rightarrow 27 = 3 + \frac{960}{m+1} \Rightarrow m+1 = 40 \Rightarrow m = 39$$

19. A normal is drawn to parabola $y^2 = 4x$ at (1,2) and tangent is drawn to the curve $y = e^x$ at (c, e^c) . If tangent and normal intersect at x-axis, then the value of c is

Ans. 04.00

Sol. For (1,2) of $y^2 = 4x \Rightarrow t = 1, a = 1$

Normal to the parabola at t is $tx + y = 2at + at^3$

$$\Rightarrow x + y = 3 \text{ intersect x-axis at } (3,0)$$

$$y = e^x \Rightarrow \frac{dy}{dx} = e^x$$

Equation of tangent to $y = e^x$ at (c, e^c) is $\Rightarrow y - e^c = e^c(x - c)$ ------(1)

$$\text{Equation (1) passes through } (3,0) \Rightarrow 0 - e^c = e^c(3 - c) \Rightarrow c = 4$$

20. The number of 3 digit numbers such that the sum of their digits is 10, is

Ans. 55.00

Sol. Let xyz be the three digit number

$$x + y + z = 10, \text{ where } x \geq 1, y \geq 0, z \geq 0$$

$$\Rightarrow (x-1) + y + z = 9, \text{ where } x-1 \geq 0, y \geq 0, z \geq 0$$

Number of non-negative integral solutions of the above equation = $n+r-1 \text{ } ^{c_{r-1}} = (9+3-1) \text{ } ^{c_{(3-1)}} = 11 \text{ } ^{c_2} = 55$

21. If $\sum_{i=1}^{10} (x_i - p) = 3$ and $\sum_{i=1}^{10} (x_i - p)^2 = 9$, then the standard deviation of x_1, x_2, \dots, x_{10} is

Ans: (00.90)

Sol:
$$S.D = \sqrt{\frac{\sum_{i=1}^{10} (x_i - p)^2}{10} - \left(\frac{\sum_{i=1}^{10} (x_i - p)}{10}\right)^2} = \sqrt{\frac{9}{10} - \frac{9}{100}} = 0.9$$

22. Let S be the set of all integral solutions (x, y, z) of the system of equations $x - 2y + 5z = 0$, $-2x + 4y + z = 0$ and $-7x + 14y + 9z = 0$ such that $15 \leq x^2 + y^2 + z^2 \leq 150$, then the number of elements in the set S is equal to

Ans: 08.00

Sol: Given system of equations

$$x - 2y + 5z = 0 \text{ -----(1)}$$

$$-2x + 4y + z = 0 \text{ -----(2)}$$

$$-7x + 14y + 9z = 0 \text{ -----(3)}$$

$$(1)*2+(2) \Rightarrow 11z = 0 \Rightarrow z = 0$$

From (1), $x = 2y$

Also given, $15 \leq x^2 + y^2 + z^2 \leq 150$

$$\Rightarrow 15 \leq 5y^2 \leq 150$$

$$\Rightarrow 3 \leq y^2 \leq 30$$

$$\Rightarrow \sqrt{3} \leq |y| \leq \sqrt{30}$$

$$\Rightarrow y = -2, -3, -4, -5, 2, 3, 4, 5.$$

