41 YEARS OF EXCELLENCE

SRAMAYEVA JAYATHE

NARAYANA EDUCATIONAL INSTITUTIONS

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SRAMAYEVA JAYATHE

THE NARAYANA GROUP

(Memory Based)
JEE MAIN – 2020
September Session
04-09-2020 (Shift-I)
(MATHS)

PART :: MATHS

1. In a group 63% people read news paper A while 76% people new paper B. If x% people read both A and B then x may be

1) 37% 2) 68% 3) 29% 4) 55%

Ans. (4)

Sol. $n(A)=63\%$

$n(B)=76\%$

$n(A\cap B)=x\%$

Let $n(\cup)=100$

$n(A)=63, n(B)=76, n(A\cap B)=x$

$n(A\cup B)=n(A)+n(B)-n(A\cap B)\le 100$

$63+76-x\le 100$

$x\ge 39$

But $n(A\cap B)\le n(A)$ $\quad\therefore 39\le x\le 63$

2. If $f(x)=\int\frac{\sqrt{x}}{(1+x)}dx,$ then find the value of $f(3)-f(1)$

1) $\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ 2) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$ 3) $\frac{\pi}{12}+\frac{1}{3}-\frac{\sqrt{3}}{4}$ 4) $\frac{\pi}{12}+\frac{1}{4}-\frac{\sqrt{3}}{4}$

Ans. (2)

Sol. $f(x)=\int\frac{\sqrt{x}}{(1+x)^2}dx$

Let $x=\tan^2\theta$

$dx=2\tan\theta\sec^2\theta\, d\theta$

$\int\frac{\tan\theta}{\left(1+\tan^2\theta\right)^2}2\tan\theta\sec^2\theta\ d\theta$

$f(x)=\int\frac{\tan\theta}{\sec^4\theta}2\tan\theta\sec^2\theta\ d\theta$

$f(x)=\int 2\tan^2\theta\cos^2\theta\, d\theta$

$f(x)=\int 2\sin^2\theta\ d\theta$

$f(x)=\int(1-\cos 2\theta)\ d\theta$

$f(x)=\theta-\frac{\sin 2\theta}{2}+C=\theta-\frac{\tan\theta}{1+\tan^2\theta}+C$

$f(x)=\tan\sqrt{x}-\frac{\sqrt{x}}{1+x}+C$

Now $f(3)-f(1)=\tan\sqrt{3}-\frac{\sqrt{3}}{1+3}-\tan^{-1}\sqrt{1}+\frac{1}{1+1}$

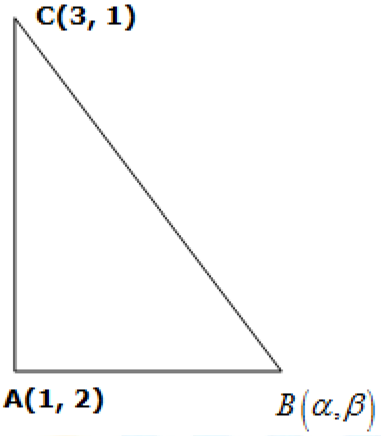
$=\frac{\pi}{3}-\frac{\pi}{4}+\frac{1}{2}-\frac{\sqrt{3}}{4}=\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

3. Let ΔABC is a right angled triangle right angled at A such that $A(1,2),C(3,1)$ and area of $\Delta ABC=5\sqrt{5}$ then abscissa of B can be

1) $1+5\sqrt{2}$ 2) $1+2\sqrt{5}$ 3) $1-5\sqrt{2}$ 4) $3+2\sqrt{5}$

Ans. (2)

Sol.



$m_{AB}=\frac{\beta-2}{\alpha-1}$

$m_{AC}=\frac{2-1}{1-3}=-\frac{1}{2}$

$AB\perp AC \quad \therefore \frac{\beta-2}{\alpha-1}\left(-\frac{1}{2}\right)=-1$

$\beta=2\alpha-2+2\Rightarrow\beta=2\alpha$

Now area of $\Delta ABC=5\sqrt{5}=\frac{1}{2}AB.AC$

$\Rightarrow\frac{1}{2}\sqrt{(3-1)^2+(1-2)^2}.\sqrt{(\alpha-1)^2+(\beta-2)^2}=5\sqrt{5}$

$\Rightarrow\sqrt{(\alpha-1)^2+(2\alpha-2)^2}=10$

$\Rightarrow\sqrt{(\alpha-1)^2}\sqrt{5}=10 \quad \Rightarrow|\alpha-1|=2\sqrt{5} \Rightarrow \alpha=1\pm2\sqrt{5}$

4. Let $f(x)=|x-2|$ and $g(x)=f(f(x)),\ x\in[0,4]$, then $\int_0^3(g(x)-f(x))dx=$

1) 1 2) 2 3) 3 4) 4

Ans. (1)

Sol. $f(x)=|x-2|=\begin{cases}2-x & x<2\\ x-2 & x\ge 2\end{cases}$

$g(x)=f(f(x))=\begin{cases}2-f(x) & f(x)<2\\ f(x)-2 & f(x)\ge 2\end{cases}$

$=\begin{cases}2-(2-x) & 2-x<2,\quad x<2\\ (2-x)-2 & 2-x\ge 2,\quad x<2\\ 2-(x-2) & x-2<2,\quad x\ge 2\\ (x-2)-2 & x-2\ge 2,\quad x\ge 2\end{cases}$

$=\begin{cases}x & 0<x<2\\ -x & x\le 0\\ 4-x & 2\le x<4\\ x-4 & x\ge 4\end{cases}$

$\int_0^3 (g(x)-f(x))dx=\int_0^2 x\,dx+\int_0^3 (4-x)dx-\int_0^3 |x-2|dx=1$

5. $\sum_{r=0}^{20} {}^{50-r}C_6=$

1) ${}^{51}C_7-{}^{30}C_6$ 2) ${}^{51}C_6-{}^{30}C_6$ 3) ${}^{51}C_7-{}^{30}C_7$ 4) ${}^{51}C_6-{}^{30}C_7$

Ans. (3)

Sol. $\sum_{r=0}^{20} {}^{50-r}C_6=$

$={}^{50}C_6+{}^{49}C_6+{}^{48}C_6+\dots\dots\dots+{}^{30}C_6=-{}^{30}C_7+{}^{30}C_7+{}^{30}C_6+{}^{31}C_6+{}^{32}C_6+\dots\dots+{}^{50}C_6$

$=-{}^{30}C_7+{}^{31}C_7+{}^{31}C_6+{}^{32}C_6+\dots\dots+{}^{50}C_6$

$=-{}^{30}C_7+{}^{32}C_7+{}^{32}C_6+\dots\dots+{}^{50}C_6$

$=-{}^{30}C_7+{}^{51}C_7$

6. Let $x\frac{dy}{dx}-y=x^2(x\cos x+\sin x)$ is a differential equation. If $f(\pi)=\pi$ then $f''\left(\frac{\pi}{2}\right)+f\left(\frac{\pi}{2}\right)=$

1) $\frac{\pi}{2}+2$ 2) $\frac{\pi}{2}-2$ 3) $\frac{\pi}{2}+1$ 4) $\frac{\pi}{2}-1$

Ans. (1)

Sol. Given $x\frac{dy}{dx}-y=x^2(x\cos x+\sin x)$

$\Rightarrow \frac{dy}{dx} - \frac{1}{x}y - x(\cos x + \sin x)$ $\quad\quad\therefore I.F. = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

$\therefore$ solution is $y.\frac{1}{x} = \int \frac{1}{x}.x(x\cos x + \sin x)dx + C$

$\frac{y}{x} = \int (x\cos x + \sin x)dx + C$

$\frac{y}{x} = x\sin x + C,\ \frac{\pi}{\pi} = 0 + C \quad \Rightarrow C = 1$

$y = x^2 \sin x + x$

$\frac{dy}{dx} = x^2\cos x + 2x\sin x + 1$

$\frac{d^2y}{dx^2} = -x^2\sin x + 2x\cos x + 2x\cos x + 2\sin x = -x^2\sin x + 4x\cos x + 2\sin x$

$f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = \left(-\frac{\pi^2}{4} + 4.0 + 2\right) + \left(\frac{\pi^2}{4}.1 + \frac{\pi}{2}\right)$

$= -\frac{\pi^2}{4} + 2 + \frac{\pi^2}{4} + \frac{\pi}{2}$

$= \frac{\pi}{2} + 2$

7. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be an ellipse such that $LR = 10$ and its eccentricity is equal to maximum value of quadratic expression $f(t) = \frac{5}{12} + t - t^2$ then $(a^2 + b^2) =$

Ans. 126

Sol. $LR = \frac{2b^2}{a} = 10 \quad\quad \Rightarrow b^2 = 5a$

$f(t) = \frac{5}{12} - \left(t^2 - t + \frac{1}{4} - \frac{1}{4}\right) = \frac{5}{12} + \frac{1}{4} - \left(t - \frac{1}{2}\right)^2$

$= \frac{2}{3} - \left(t - \frac{1}{2}\right)^2$

Maxi $f(t) = \frac{2}{3} = e$

$b^2 = a^2(1 - e^2)$

$5a = a^2\left(1 - \frac{4}{9}\right) \Rightarrow \quad 5 = \frac{5}{9}a \ \Rightarrow a^2 = 81, b^2 = 45$

$a^2 + b^2 = 126$

8. If α, β are roots of $x^2 - 3x + p = 0$ and γ, δ are roots of $x^2 - 6x + q = 0$ and $\alpha, \beta, \gamma, \delta$ are in increasing geometric progression then value of $\frac{2q+p}{2q-p}$ is equal to

1) $\frac{7}{9}$ 2) $-\frac{7}{9}$ 3) $\frac{9}{7}$ 4) $-\frac{9}{7}$

Ans. (3)

Sol. $\alpha = a, \beta = ar, \gamma = ar^2, \delta = ar^3$

$\alpha + \beta = 3 \Rightarrow a + ar = 3 \quad ....(1)$

$\gamma + \delta = 6 \Rightarrow ar^2 + ar^3 = 6 \quad ....(2)$

By (1) and (2) $\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{6}{3} \Rightarrow r^2 = 2$

$r = \sqrt{2}$

$\therefore \frac{2q+p}{2q-p} = \frac{9}{7}$

9. The mean and variance of 5, 7, 12, 10, 15, 14, a, b are 10 and 13.5 respectively then value of $|a-b| =$

1) 5 2) 6 3) 7 4) 8

Ans. (3)

Sol. $\frac{5+7+12+10+15+14+a+b}{8} = 10$

$\Rightarrow 63 + a + b = 80 \Rightarrow a + b = 17 \quad ....(1)$

$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i^2}{n}\right)^2$

$\Rightarrow 13.5 = \frac{25+49+144+100+196+a^2+b^2}{8} - 100$

$908 = a^2 + b^2 + 739$

$a^2 + b^2 = 169$

$(a+b)^2 - 2ab = 169$

$289 - 169 = 2ab \Rightarrow ab = 60$

$\therefore |a-b|^2 = (a+b)^2 - 4ab = 289 - 240 = 49$

$\therefore |a-b| = 7$

10. If $1 + (1 - 2^2.1) + (1 - 4^2.3) + (1 - 6^2.5) + ... + (1 - 20^2.19) = \alpha - 220\beta$ then $(\alpha, \beta) =$

1) $(11, 97)$ 2) $(11, 103)$ 3) $(10, 97)$ 4) $(10, 103)$

Ans. (2)

Sol. $S=1+\sum_{r=1}^{10}1-(2r)^2(2r-1)=1+10-\sum_{r=1}^{10}\left(8r^3-4r^2\right)=11-\left[8\left(\frac{10\times11}{2}\right)^2-4.\left(\frac{10\times11\times21}{6}\right)\right]$

$=11-\left[2(110)^2-140\times11\right]$

$=11-22(1100-70)$

$=11-220(110-7)$

$\therefore \alpha-220\beta$

$=11-220(103)$

$\therefore\ \alpha=11,\ \beta=103$

$(\alpha,\beta)=(11,103)$

11. For equation $[x]^2+2[x+2]-7=0,\ x\in R$ number of solution of equation is / are

(1) for inter solution (2) infinite solution

(3) No solution (4) two solution

Ans. (2)

Sol. $[x]^2+2[x+2]-7=0$

$[x]^2+2([x]+2)-7=0$

Let $[x]=t$

$t^2+2t-3=0$

$t=1,-3$

$[x]=-3,1$

$x\in[-3,-2)\cup[1,2)$

Hence infinite solution

12. Integration: $\int\frac{x^2}{(x\sin x+\cos x)^2}dx$ is equal to

1) $\frac{\sin x+x\cos x}{x\sin x+\cos x}+C$ 2) $\frac{\sin x+x\cos x}{x\sin x-\cos x}+C$

3) $\frac{x\cos x-\sin x}{\cos x-x\sin x}+C$ 4) $\frac{\sin x-x\cos x}{x\sin x+\cos x}+C$

Ans. (4)

Sol. $\int\frac{x^2\ dx}{(x\sin x+\cos x)^2}=\int\frac{x}{\cos x}.\frac{x\cos x}{(x\sin x+\cos x)^2}dx$

$=\frac{x}{\cos x}\int\frac{x\cos x}{(x\sin x+\cos x)^2}-\int\left[\frac{d}{dx}(x\sec x)\int\frac{x\cos x}{(x\sin x+\cos x)}dx\right]dx$

$$= \frac{x}{\cos x}\left(-\frac{1}{x\sin x+\cos x}\right)+\int \sec^2 x\,dx$$

$$\left(\because \int \frac{x\cos x}{(x\sin x+\cos x)^2}dx = \frac{-1}{x\sin x+\cos x} \quad and\right.$$

$$\left.\frac{d}{dx}(x\sec x) = \sec x + x\sec x\tan x = \sec x\left(1+\frac{x\sin x}{\cos x}\right) = \sec^2 x(x\sin x+\cos x)\right)$$

$$\frac{d}{dx}(x\sec x) = \sec x + x\sec x\tan x = \sec x\left(1+\frac{x\sin x}{\cos x}\right) = \sec^2 x(x\sin x+\cos x)$$

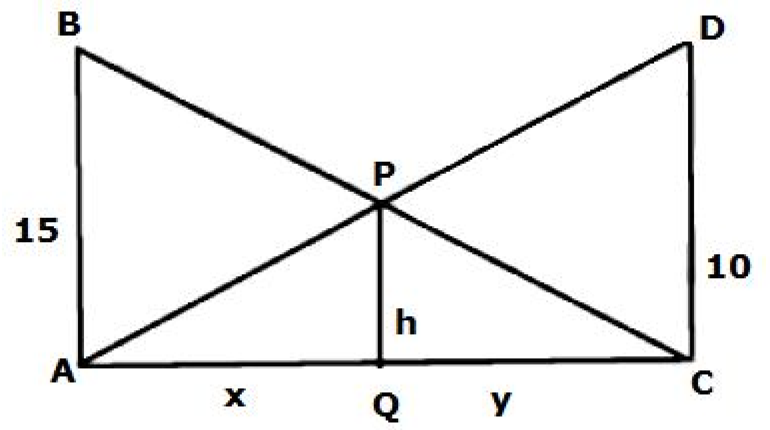
$$= \frac{-x}{\cos x(x\sin x+\cos x)}+\frac{\sin x}{\cos x}+C$$

$$= \frac{-x+x\sin^2 x+\sin x\cos x}{\cos x(x\sin x+\cos x)} = \frac{\sin x - x\cos x}{x\sin x+\cos x}+C$$

13. Two poles AB and CD of height 15 m and 10 m respectively. A and C are on level ground, Point of intersection of AD and BC is P then height of P is

Ans. 6

Sol.



$$\Delta AQP \sim \Delta ACD \quad \Rightarrow \frac{x}{h} = \frac{x+y}{10} \quad ....(1)$$

$$\because \quad \Delta CQP \sim \Delta CAB \quad \Rightarrow \frac{y}{h} = \frac{x+y}{15} \quad ....(2)$$

$$(1)+(2) \rightarrow \frac{x+y}{h} = (x+y)\left(\frac{1}{10}+\frac{1}{15}\right) \Rightarrow h = 6$$

14. Consider two statements

$S_1 :\sim p \rightarrow (\sim q \leftrightarrow \sim p)$ is a tautology

$S_2 : (\sim q \wedge p) \rightarrow q$ is fallacy then

(1) Statement I is true, Statement II is false

(2) Statement I is false, Statement II is true

(3) Both true

4) Both false

Ans. (4)

Sol. $I :\sim p \to (\sim q \leftrightarrow \sim p)$

p	q	~p	~q	$\sim q \leftrightarrow \sim p$	(I) $\sim p(\sim q \leftrightarrow \sim p)$	$p \wedge \sim q$	(II) $(\sim q \wedge p) \to q$
T	T	F	F	T	T	F	T
T	F	F	T	F	T	T	F
F	T	T	F	F	F	F	T
F	F	T	T	T	T	F	T

Both are false

15. If $u = \dfrac{2z+i}{z-ki}$ where $z = x+iy$ and $k>0$

Curve $\text{Re}(u)+\text{Im}(u)=1$ cuts y-axis at two point P and Q such that PQ=5 then value k is

1) 1 2) 2 3) 3 4) 4

Ans. (2)

Sol. $u = \dfrac{2(x+iy)+i}{(x+iy)-ki} = \dfrac{2x+(2y+1)i}{x+(y-k)i} \times \dfrac{x-(y-k)i}{x+(y-k)i}$

Real part of $u = \text{Re}(u) = \dfrac{2x^2+(2y+1)(y-k)}{x^2+(y-k)^2}$

Imaginary part of $u = \text{Im}(u) = \dfrac{x(2y+1)-2x(y-k)}{x^2+(y-k)^2}$

Now $\text{Re}(u)+\text{Im}(u)=1$

$$\frac{2x^2+(2x+1)(y-k)+x(2y+1)-2x(y-k)}{x^2+(y-k)^2}=1$$

For y-axis put $x=0 \Rightarrow \dfrac{(2y+1)(y-k)}{(y-k)^2}=1$

$\Rightarrow (2y+1)(y-k)=(y-k)^2$

$\Rightarrow (y-k)(y+(1+k))=0$

$y=k,-(1+k)$

Now point P (0, k), $Q(0,-(1+k))$

$PQ = |2k+1| = 5$

$2k+1=\pm 5$

$2k=4,-6$

$k=2,-3$

Hence $k=2 \quad (k>0)$

16. Probability of hitting a target is $\frac{1}{10}$ then find the maximum number of trails so that probability of at least one success is greater than $\frac{1}{4}$ is

Ans. 3

Sol. $p=\frac{1}{10}, q=\frac{9}{10}$

p (not hitting in n trails) $=\left(\frac{9}{10}\right)^n$

$\therefore p$ (at least one hit) $1-\left(\frac{9}{10}\right)^n \geq \frac{1}{4}$

$\Rightarrow \left(\frac{9}{10}\right)^n \leq \frac{3}{4}$

$\Rightarrow (9)^n \leq 75$

$n=3 \Rightarrow 0.729 \leq .75$ which is true

17. Let $A=\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}, 0<\theta<\frac{\pi}{24}$ and $A^5=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then which statement is false

1) $a^2-b^2=\frac{1}{2}$ 2) $a^2+b^2 \in (0,1)$ 3) $a^2-d^2=0$ 4) $a^2-c^2=1$

Ans. (1)

Sol. $A^2=\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}=\begin{bmatrix} \cos^2\theta-\sin^2\theta & 2i\sin\theta\cos\theta \\ 2i\sin\theta\cos\theta & \cos^2\theta-\sin^2\theta \end{bmatrix}=\begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{bmatrix}$

$A^3=\begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}=\begin{bmatrix} \cos 3\theta & i\sin 3\theta \\ i\sin 3\theta & \cos 3\theta \end{bmatrix}$

$A^5=\begin{bmatrix} \cos 5\theta & i\sin 5\theta \\ i\sin 5\theta & \cos 5\theta \end{bmatrix}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$a=\cos 5\theta, b=i\sin 5\theta, c=i\sin 5\theta, d=\cos 5\theta$

$(1) a^2-b^2=\cos^2 5\theta+\sin^2 5\theta=1$

$(2) a^2+b^2=\cos^2 5\theta-\sin^2 5\theta=\cos 10\theta \in (0,1)$ as $0<\theta<\frac{\pi}{24}$

(3) $(3) a^2-d^2=0$

(4) $(4) a^2-c^2=a^2-b^2=1$

18. If $\left(a-\sqrt{2}b\cos x\right)\left(a+\sqrt{2}b\cos y\right)=a^2-b^2$ then value of $\frac{dy}{dx}$ at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ is

1) $\frac{a-b}{a+b}$ 2) $\frac{a+b}{a-b}$ 3) $\frac{2a+b}{a-b}$ 4) $\frac{2a-b}{a-b}$

Ans. (2)

Sol. $\left(a-\sqrt{2}b\cos x\right)\left(-\sqrt{2}b\sin y\right)\frac{dy}{dx}+\sqrt{2}b\sin x\left(a+\sqrt{2}\cos y\right)=0$

$$\frac{dy}{dx}=\frac{\sqrt{2}b\sin x\left(a+\sqrt{2}b\cos y\right)}{\sqrt{2}b\sin y\left(a-\sqrt{2}b\cos x\right)}=\frac{\sin x\left(a+\sqrt{2}b\cos y\right)}{\sin y\left(a-\sqrt{2}b\cos x\right)}$$

$$\left.\frac{dy}{dx}\right|_{\left(\frac{\pi}{4},\frac{\pi}{4}\right)}=\frac{a+b}{a-b}$$

19. $f\left(x+y\right)=f\left(x\right)+f\left(y\right)+xy^2+x^2y$ and $\lim\limits_{x\to 0}\frac{f\left(x\right)}{x}=1,$ then find value of $f'\left(3\right)$

Ans. 10

Sol. $f\left(x+y\right)=f\left(x\right)+f\left(y\right)+xy^2+x^2y$

$f'\left(x+y\right)=f'\left(x\right)+0+y^2+2xy$

Put $y=-x$

$f'\left(0\right)=f'\left(x\right)+0+x^2-2x^2$

$1=f'\left(x\right)-x^2$

$f'\left(x\right)=1+x^2$

$f'\left(3\right)=10$

20. If f is twice differentiable function for $x\in R$ such that $f\left(2\right)=5, f'\left(2\right)=8$ and $f'\left(x\right)\ge 1, f''\left(x\right)\ge 4$ then

1) $f\left(5\right)+f'\left(5\right)\le 26$
2) $f\left(5\right)+f'\left(5\right)\ge 28$
3) $f\left(5\right)+f'\left(5\right)\le 28$
4) None of these

Ans. (2)

Sol. Given

$\Rightarrow\left(f\left(x\right)_2^5\right)\ge\left(x\right)_2^5 \quad \Rightarrow f\left(5\right)-f\left(2\right)\ge 3 \quad \Rightarrow f\left(5\right)\ge 8 \quad ....(1)$

Now $f''\left(x\right)\ge 4\Rightarrow\int_2^5 f''\left(x\right)dx\ge\int_2^5 4\,dx$

$=\left(f'\left(x\right)\right)_2^5\ge\left(4x\right)_2^5$

$\Rightarrow f'\left(5\right)-f'\left(2\right)\ge 12$

$\Rightarrow f'\left(5\right)\ge 20 \quad ....(2)$

(1) + (2) $\Rightarrow f\left(5\right)+f'\left(5\right)\ge 28$

21. If $\left(2x^2+3x+4\right)^{10}=\sum_{r=0}^{20} a_r x^r$, then $\frac{a_7}{a_{13}}=$

Ans. 8

Sol. General term $\frac{10!}{r_1!r_2!r_3!}\left(2x^2\right)^{r_1}(3x)^{r_2}(4)^{r_3}$

$$a_7=\frac{10!.2^3.3.4^6}{3!1!6!}+\frac{10!.2^2.3^3.4^5}{2!3!5!}+\frac{10!.2.3^5.4^4}{1!5!4!}+\frac{10!.3^7.4^3}{7!3!}$$

$$a_{13}=\frac{10!.2^6.3.4^3}{6!1!3!}+\frac{10!.2^5.3^3.4^2}{5!3!2!}+\frac{10!.2^4.3^5.4^1}{4!5!1!}+\frac{10!2^3.3^7.4^0}{3!7!}$$

$$\frac{a_7}{a_3}=2^3=8$$

22. If form a point (3, 3) on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ a normal is drawn which cuts x axis at $(9,0)$ then value of $\left(a^2,e^2\right)$ is

1) $\left(\frac{9}{2},3\right)$ 2) $\left(\frac{9}{2},1\right)$ 3) $(9,3)$ 4) $(3,9)$

Ans. (1)

Sol. hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$P(3,3)$ lies on hyperbola then $\frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{9}$ ....(1)

Normal at $(3,3)$ is

$$\frac{a^2x}{3}+\frac{b^2y}{3}=a^2+b^2$$

Pass through $(9,0)$

$$3a^2=a^2+b^2\Rightarrow 2a^2=b^2$$

Then $\frac{1}{a^2}-\frac{1}{2a^2}=\frac{1}{9}$

$2a^2=9\Rightarrow a^2=\frac{9}{2}$ and $b^2=9$

$$e^2=1+\frac{b^2}{a^2}1+2=3$$


thank
you