

## NARAYANA'S SENSATIONAL SUCCESS ACROSS INDIA

### 7 Students Secured 100 Percentile in All India JEE Main-2020



### ADMISSIONS OPEN (2020-21)

### OUR REGULAR CLASSROOM PROGRAMME

**One Year Classroom Program  
JEE/NEET-2021**  
(for students moving from XI to XII)

**Two Year Classroom Program  
JEE/NEET-2022**  
(for students moving from X to XI)

**Three Year Integrated Classroom Program  
JEE/NEET-2023**  
(for students moving from IX to X)

**Four Year Integrated Classroom Program  
JEE/NEET-2024**  
(for students moving from VIII to IX)

**FOUNDATION PROGRAMMES  
For NTSE, NSEJS, JSTSE,  
Olympiads & School/Board Exams**  
(for students moving to  
Class VI, VII, VIII, IX & X)

**APEX BATCH  
Two years school Integrated  
Classroom Program - 2022**  
For JEE Main & Advance / NEET (for XI Studying Students)  
Course Feature : Complete Coverage of CBSE-Regular Classes-Weekly Test & Regular Analysis-Lab Facility  
Motivation & Counseling-Competitive Exam Prep.-Ample time for self study

**Online Classes for IIT/NEET/Foundation/Olympiads**

- Access Recording of Past Classes on n-Learn App
- Online Parent Teacher Meeting
- Personalized Extra Classes & Live Doubt Solving
- Hybrid/Customized Classroom model
- Video Solution of Weekly/Fortnightly Test
- Printed Study Material will be sent by us
- n-Learn App
- Counselling Motivational sessions
- Affordable Fee
- Doubt Classes / Practice Classes
- Provision to Convert from online to regular classroom programme
- Once Classes resume by just paying nominal fee

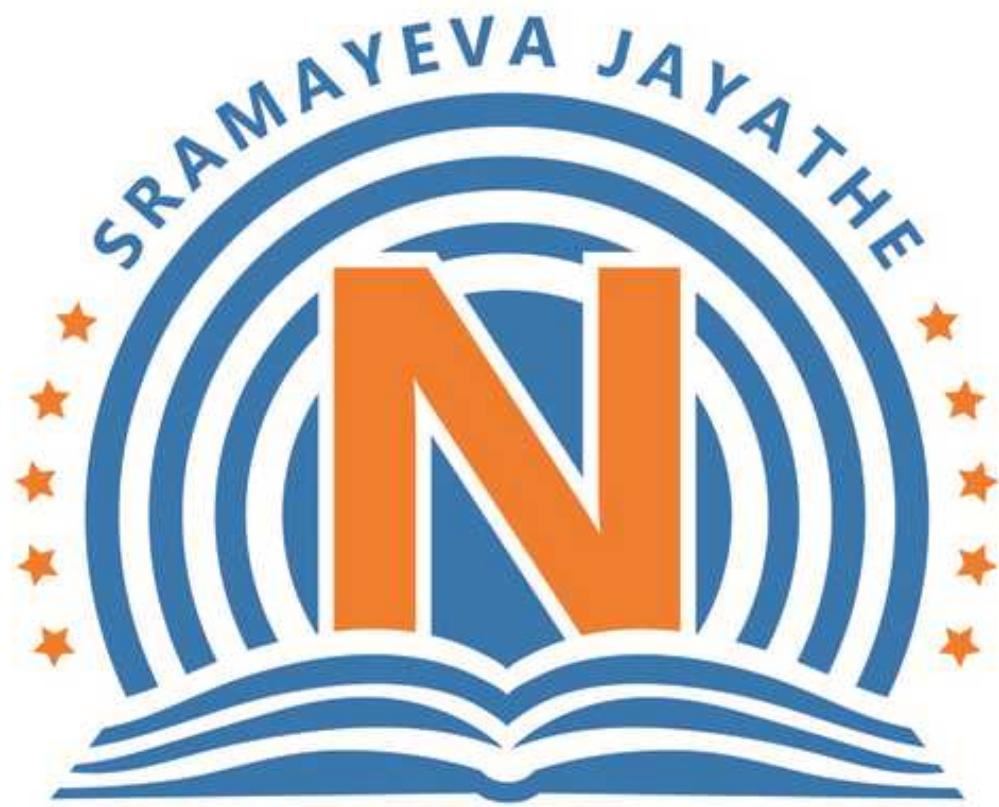
**Online Test**

- Micro & Macro Analysis
- Relative performance (All India Ranking)
- Question wise Analysis
- Unlimited Practice Test
- Grand Test

**NARAYANA**

**Digital Classes**  
STUDY ONLINE FROM HOME

For Class  
**7<sup>th</sup> to 12<sup>th</sup> +**



THE NARAYANA GROUP

**JEE-MAIN-2021**

**MARCH ATTEMPT**

**16.03.21 SHIFT - II**

**MATHEMATICS**

# MATHEMATICS



**Ans.** (4)

**Sol.** Taking sine both sides

$$\begin{aligned}
 & \frac{3x}{5} \sqrt{1 - \frac{16x^2}{25}} + \frac{4x}{5} \sqrt{1 - \frac{9x^2}{25}} = x \\
 \Rightarrow & 3x \sqrt{25 - 16x^2} = 25x - 4x \sqrt{25 - 9x^2} \\
 \Rightarrow & x = 0 \text{ or } 3\sqrt{25 - 16x^2} = 25 - 4\sqrt{25 - 9x^2} \\
 \Rightarrow & 9(25 - 16x^2) = 625 - 200\sqrt{25 - 9x^2} + 16(25 - 9x^2) \\
 \Rightarrow & 200\sqrt{25 - 9x^2} = 800 \\
 \Rightarrow & \sqrt{25 - 9x^2} = 4 \\
 \Rightarrow & x^2 = 1 \\
 \Rightarrow & x \pm 1 \\
 \therefore & \text{Total number of solution} = 3
 \end{aligned}$$

$\therefore$  Total number of solution = 3

2.

Number of elements	Group-1	Group-2
Number of elements	10	n
Mean	2	3
variance	2	1

If combined variance of both groups is  $\frac{17}{9}$ , then find 'n'

**Ans. (5)**

**Sol.** For group-1 :  $\frac{\sum x_i}{10} = 2 \Rightarrow \sum x_i = 20$

$$\frac{\sum x_i^2}{10} - (2)^2 = 2 \Rightarrow \sum x_i^2 = 60$$

$$\text{for group-2 : } \frac{\sum y_i}{n} = 3 \Rightarrow \sum y_i = 3n$$

$$\frac{\sum y_i^2}{n} - 3^2 = 1 \Rightarrow \sum y_i^2 = 10n$$

Now, combined variance

$$\sigma^2 = \frac{\sum(x_i^2 + y_i^2)}{10+n} - \left( \frac{\sum(x_i + y_i)}{10+n} \right)^2$$

$$\Rightarrow \frac{17}{9} = \frac{60+10n}{10+n} - \frac{(20+3n)^2}{(10+n)^2}$$

$$\Rightarrow 17(n^2 + 20n + 100) = 9(n^2 + 40n + 200)$$

$$\Rightarrow 8n^2 - 20n - 100 = 0$$

$$\Rightarrow 2n^2 - 5n - 25 = 0 \Rightarrow n = 5$$

3. If  $f(x) = \begin{cases} \frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x \neq 0 \\ \alpha, & x = 0 \end{cases}$

then find  $\alpha$  if  $f(x)$  is continuous at  $x = 0$  (Here  $\{x\}$  denotes fractional part of  $x$ )

- (1)  $\frac{\pi}{4}$       (2)  $\frac{\pi}{\sqrt{2}}$       (3)  $\pi$       (4) no value of  $\alpha$

**Ans.** (4)

**Sol.**  $RHL = \lim_{x \rightarrow 0^+} \frac{\cos^{-1}(1-x^2)\sin^{-1}(1-x)}{x(1-x^2)} = \frac{\pi}{2} \lim_{x \rightarrow 0^+} \frac{\cos^{-1}(1-x^2)}{x}$   
 $= \frac{\pi}{2} \lim_{x \rightarrow 0^+} \frac{-1}{\sqrt{1-(1-x^2)^2}} (-2x) \quad (\text{L'Hospital Rule})$   
 $= \pi \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{2x^2-x^4}} = \pi \lim_{x \rightarrow 0^+} \frac{1}{\sqrt{2-x^2}} = \frac{\pi}{\sqrt{2}}$   
 $LHL = \lim_{x \rightarrow 0^-} \frac{\cos^{-1}(1-(1+x)^2)\sin^{-1}(-x)}{(1+x)-(1+x)^3} = \frac{\pi}{2} \lim_{x \rightarrow 0^-} \frac{\sin^{-1} x}{(1+x)[(1+x)^2-1]} = \frac{\pi}{2} \lim_{x \rightarrow 0^-} \frac{\sin^{-1} x}{x^2+2x}$   
 $= \frac{\pi}{2} \left(\frac{1}{2}\right) = \frac{\pi}{4}$

As  $LHL \neq RHL$  so  $f(x)$  is not continuous at  $x = 0$

4. Let the circle  $x^2 + y^2 - 9x - 2ay + c = 0$  has  $x$ -intercept and  $y$ -intercept respectively  $2\sqrt{2}$  and  $2\sqrt{5}$ . Find distance of farthest tangent from origin which is perpendicular to  $2y + x = 0$   
 (1)  $\sqrt{6}$       (2)  $\sqrt{30}$       (3)  $\sqrt{10}$       (4)  $2\sqrt{6}$

**Ans.** (1)

**Sol.**  $2\sqrt{\frac{a^2}{4}-c} = 2\sqrt{2}, \quad 2\sqrt{a^2-c} = 2\sqrt{5}$

$$\frac{a^2}{4}-c=2 \quad a^2-c=5$$

$$c=\frac{a^2}{4}-2$$

$$a^2=4c+8$$

$$3c+8=5$$

$$c=-1, a^2=4$$

Equation of tangent

$$y=2x+c'$$

$$D = \left| \frac{c'}{\sqrt{5}} \right| = r$$

$$\frac{c}{\sqrt{5}} = \sqrt{\frac{a^2}{4} + a^2 - c}$$

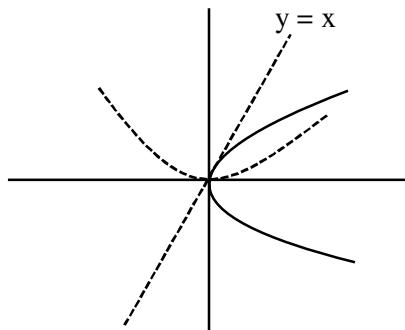
$$\frac{c}{\sqrt{5}} = \sqrt{1+5} = \sqrt{30}$$

$$D = \sqrt{6}$$

5. Let image of  $y^2 = 4x$  in  $y = x$  is a parabola. The equation of tangent from  $(2, 1)$  to new parabola is

$$(1) y = x + 1 \quad (2) x = y + 1 \quad (3) y = 2x - 3 \quad (4) y = \frac{x}{2} + 1$$

**Ans.** (2)



**Sol.**

Image of  $y^2 = 4x$  w.r.t.  $y = x$  is  $x^2 = 4y$

tangent from  $(2, 1)$

$$xx_1 = 2(y + y_1)$$

$$2x = 2(y + 1)$$

$$x = y + 1$$

6. If point of intersection of the curves  $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$  and  $x^2 + y^2 = 4b$ , where  $b > 4$ , lies on the curve

$y^2 = 3x^2$  then find 'b'

$$(1) 4 \quad (2) 12 \quad (3) 10 \quad (4) 6$$

**Ans.** (2)

$$\frac{x^2}{16} + \frac{y^2}{b^2} = 1 \quad \dots\dots(1)$$

$$x^2 + y^2 = 4b \quad \dots\dots(2)$$

$$y^2 = 3x^2 \quad \dots\dots(3)$$

From eq (2) and (3)  $x^2 = b$  and  $y^2 = 3b$

$$\begin{aligned} \text{from equation (1)} \quad & \frac{b}{16} + \frac{3b}{b^2} = 1 \\ & \Rightarrow b^2 + 48 = 16b \\ & \Rightarrow b = 12 \end{aligned}$$

7. Find the area bounded between the curve whose differential equation are given by

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \text{ and } \frac{dx}{dy} = \frac{x^2 - y^2}{2xy}$$

Given that both the curve passes through the point  $(1, 1)$

$$(1) (\pi + 1) \quad (2) (\pi - 1) \quad (3) \left(\frac{\pi}{2} - 1\right) \quad (4) \left(\frac{\pi}{2} + 1\right)$$

**Ans.** (3)

**Sol.**  $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Put  $y = vx$

$$v + x \frac{dv}{dx} = \frac{v^2 x^2 - x^2}{2vx^2} = \frac{v^2 - 1}{2v}$$

$$x \frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v} = -\frac{(v^2 + 1)}{2v}$$

$$\Rightarrow \frac{2v}{v^2 + 1} dv = -\frac{dx}{x}$$

$$\ell n(v^2 + 1) = -\ell nx + \ell nc \Rightarrow v^2 + 1 = \frac{c}{x}$$

$$\Rightarrow \frac{y^2}{x^2} + 1 = \frac{c}{x} \Rightarrow x^2 + y^2 = cx$$

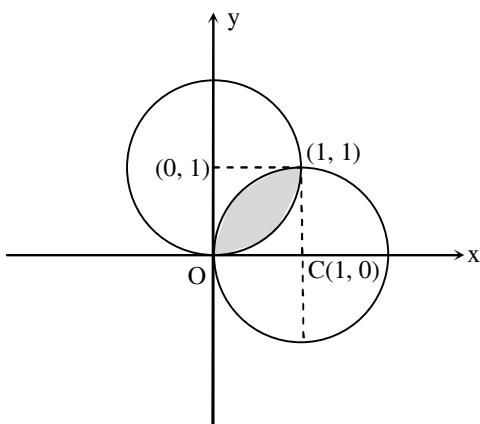
If pass through  $(1, 1)$

$$\therefore x^2 + y^2 - 2x = 0$$

similarly for second differential equation  $\frac{dx}{dy} = \frac{x^2 - y^2}{2xy}$

equation of curve is  $x^2 + y^2 - 2y = 0$

Now required area is



$$= \left( \frac{1}{4} \times \pi \times 1^2 - \frac{1}{2} \times 1 \times 1 \right) \times 2$$

$$= \left( \frac{\pi}{2} - 1 \right) \text{ sq. units}$$

8. If  $\exp \left[ \left( \frac{(|z|+3)(|z|-1)}{|z|+1} \right) \log_e 2 \right] \geq \log_{\sqrt{2}} |5\sqrt{7} + 9i|$ , then find the minimum value of  $|z|$

(4) 6

**Ans.** (1)

$$\text{Sol. } 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq 2^3 \Rightarrow \frac{(|z|+3)(|z|-1)}{|z|+1} \geq 3$$

$$\Rightarrow |z|^2 + 2|z| - 3 \geq 3|z| + 3$$

$$\Rightarrow |z|^2 - |z| - 6 \geq 0$$

$$(|z| - 3)(|z| + 2) \geq 0$$

$$|z|_{\min} = 3$$

9. Let  $f(x) : (0, \infty) \rightarrow \mathbb{R}$  is defined  $f(x + 1) = xf(x)$  &  $g(x) : \mathbb{R} \rightarrow \mathbb{R}$  is defined as

$g(x) = \log(f(x))$ , then the value of  $|g''(5) - g''(1)|$  is

$$(1) \frac{205}{144} \quad (2) \frac{144}{205} \quad (3) \frac{144}{113}$$

(4) none of these

**Ans.** (1)

$$\text{Sol. } g(x+1) = \ln(f(x+1)) = \ln(x f(x)) = \ln x + \ln(f(x))$$

$$\Rightarrow g(x+1) - g(x) = \ell n x$$

$$g'(x+1) - g'(x) = \frac{1}{x}$$

$$\Rightarrow g''(x+1) - g''(x) = -\frac{1}{32} \quad \dots\dots(i)$$

putting  $x = 1, 2, 3, 4$  in (i), we get

$$\Rightarrow g''(2) - g''(1) = -1$$

$$g''(3) - g''(2) = -\frac{1}{4}$$

$$g''(4) - g''(3) = -\frac{1}{9}$$

$$g''(5) - g''(4) = -\frac{1}{16}$$

Adding,

$$g''(5) - g''(1) = -1 - \frac{1}{4} - \frac{1}{9} - \frac{1}{16} = -\frac{205}{144}$$

$$\therefore |g''(5) - g''(1)| = \frac{205}{144}$$

- 10.** If  $y = y(x)$  the solution of the differential equation  $\frac{dy}{dx} + y \tan x = \sin x$  and  $y(0) = 0$ , then  $y\left(\frac{\pi}{4}\right)$  is equal to

(1)  $\frac{1}{2} \ln 2$       (2)  $\frac{1}{2\sqrt{2}} \ln 2$       (3)  $\frac{1}{\sqrt{2}} \ln 2$       (4)  $\ln 2$

**Ans.** (2)

**Sol.** Integrating factor =  $e^{\int \tan x dx} = \sec x$

$\therefore$  solution of the equation is

$$y \sec x = \int \sin x \times \sec x \, dx$$

$$\Rightarrow \frac{y}{\cos x} = \ln(\sec x) + c$$

$$\text{put } x = 0, c = 0$$

$$\therefore y = \cos x \ln(\sec x)$$

$$\text{put } x = \frac{\pi}{4}$$

$$y = \frac{1}{\sqrt{2}} \ln \sqrt{2} = \frac{1}{2\sqrt{2}} \ln 2$$

- 11.** If  $(x) = \begin{vmatrix} 1+\cos^2 x & \sin^2 x & \cos 2x \\ \cos^2 x & 1+\sin^2 x & \cos 2x \\ \cos^2 x & \sin^2 x & \sin 2x \end{vmatrix}$ , then maximum value of  $f(x)$

(1)  $\sqrt{5}$       (2) 5      (3)  $\sqrt{3}$       (4)  $2\sqrt{5}$

**Ans.** (1)

**Sol.**  $C_1 \rightarrow C_1 + C_2$

$$f(x) = \begin{vmatrix} 2 & \sin^2 x & \cos 2x \\ 2 & 1+\sin^2 x & \cos 2x \\ 1 & \sin^2 x & \sin 2x \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$f(x) = \begin{vmatrix} 2 & \sin^2 x & \cos 2x \\ 0 & 1 & 0 \\ 1 & \sin^2 x & \sin 2x \end{vmatrix}$$

$$f(x) = 2 \sin 2x - \cos 2x$$

$$f(x)_{\max} = \sqrt{5}$$

- 12.** The value of integral  $\int_0^{10} \frac{[x]e^{[x]}}{e^{x-1}} dx$  is equal to

(1)  $45(1-e)$       (2)  $55(e-1)$       (3)  $45(e-1)$       (4)  $45(e+1)$

**Ans.** (3)

$$\begin{aligned}
 \text{Sol. } I &= \int_0^{10} [x] e^{[x]+1-x} dx \\
 &= \int_1^2 e^{2-x} dx + \int_2^3 2.e^{3-x} dx + \int_3^4 3.e^{4-x} dx + \dots + \int_9^{10} 9.e^{10-x} dx \\
 &= -\{(1-e) + 2(1-e) + 3(1-e) + \dots + 9(1-e)\} \\
 &= 45(e-1)
 \end{aligned}$$

13. If  $S = \{0,1,2,\dots,6\}$  and a six digit number is formed from it, find the probability it is divisible by '3'.

(1)  $\frac{4}{5}$       (2)  $\frac{4}{9}$       (3)  $\frac{1}{3}$       (4)  $\frac{2}{3}$

**Ans.** (2)

$$\text{Sol. } 6 \times 6 \times 5 \times 4 \times 3 \times 2$$

$$n(5) = 6 \times 6!$$

0,1,2,3,4,5,6

$$n(\in) = \frac{6! + 2 \times 5 \times 5}{6 \times 6!}$$

$$= \frac{6+10}{36}$$

$$= \frac{4}{9}$$

- 14.** If two matrices A and B are  $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$  and  $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$  such that  $A = XB$ , where  $X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ k & 1 \end{bmatrix}$ , then find k such that  $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$  and  $(1 + k^2) b_2^2 \neq 2b_1 b_2$ .

**Ans.** 1

$$\text{Sol.} \quad \therefore A = XB$$

$$\therefore \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ k & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\Rightarrow a_1 = \frac{1}{\sqrt{3}}(b_1 - b_2) \text{ and } a_2 = \frac{1}{\sqrt{3}}(kb_1 + b_2)$$

Given that

$$a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$$

$$\Rightarrow \frac{1}{3}(b_1 - b_2)^2 + \frac{1}{3}(kb_1 + b_2)^2 = \frac{2}{3}(b_1^2 + b_2^2)$$

$$\Rightarrow (1 + k^2) b_1^2 + 2b_2^2 + 2(k - 1)b_1b_2 = 2b_1^2 + 2b_2^2$$

$$\Rightarrow (1 + k^2) b_1^2 + 2(k - 1) b_1 b_2 = 2b_1^2$$

this is only possible when  $k = 1$

15. Consider 3 points A(-1, 1), B(3, 4) and C (2, 0). The line  $y = mx$  cuts line AC and BC at points P and Q respectively. If area of  $\Delta ABC = A_1$ , and area of  $\Delta PQC = A_2$  and  $A_1 = 3A_2$ , then positive value of m is

(1) 1

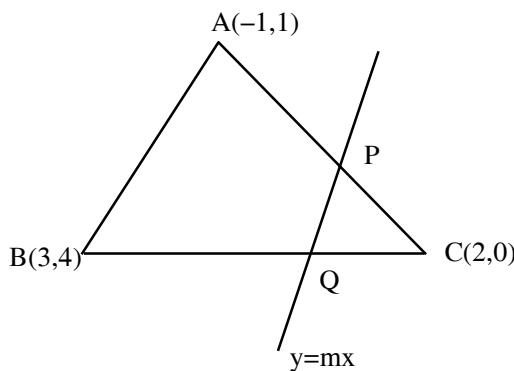
(2)  $\frac{4}{15}$

(3) 2

(4)  $\frac{15}{4}$

**Ans.** (1)

**Sol.**



$$A_1 = \Delta ABC = \frac{1}{2} \begin{vmatrix} -1 & 1 & 1 \\ 2 & 0 & 1 \\ 3 & 4 & 1 \end{vmatrix}$$

$$A_1 = \frac{13}{2}$$

$$\text{Equation of line } AC \text{ is } y - 1 = \frac{1}{3}(x + 1)$$

$$\text{solve it with line } y = mx, \text{ we get } P\left(\frac{2}{3m+1}, \frac{2m}{3m+1}\right)$$

$$\text{Equation of line } BC \text{ is } y - 0 = 4(x - 2)$$

$$\text{Solve it with line } y = mx, \text{ we get } Q\left(\frac{-8}{m-1}, \frac{-8m}{m-4}\right)$$

$$A_2 = \text{Area of } \Delta PQC = \frac{1}{2} \begin{vmatrix} 2 & 0 & 1 \\ \frac{2}{3m+1} & \frac{2m}{3m+1} & 1 \\ \frac{-8}{m-4} & \frac{-8m}{m-4} & 1 \end{vmatrix} = \frac{A_1}{3} = \frac{13}{6}$$

$$\frac{1}{2} \begin{vmatrix} 2 & 0 & 1 \\ 0 & \frac{2}{3m+1} & 1 \\ 0 & \frac{-8}{m-4} & 1 \end{vmatrix} = \frac{13}{6}$$

$$\frac{1}{2} \left| m(2) \left( \frac{2}{3m+1} + \frac{8}{m-4} \right) \right| = \frac{13}{6}$$

$$2m \left| \left( \frac{m-4+12m+4}{(3m+1)(m-4)} \right) \right| = \pm \frac{13}{6}$$

$$\frac{26m^2}{(3m+1)(m-4)} = \pm \frac{13}{6}$$

- 16.** If  $S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \dots$ . Also,  $S_{24}(x) = 1093$  and  $S_{12}(2x) = 265$ . then find a.

**Ans.** (16)

**Sol.**  $S_n(x) = (\log_a x) \underbrace{2+3+6+11+\dots}_{7}$

$$t_n = an^2 + bn + c$$

$$a + b + c = 2 \quad \dots \text{(i)}$$

$$4a + 2b + c = 3 \quad \dots \text{(ii)}$$

$$9a + 3b + c = 6 \quad \dots \text{(iii)}$$

$$\Rightarrow a = 1; b = -2; c = 3$$

$$T_n = n^2 - 2n + 3 = (n-1)^2 + 2$$

$$\therefore S_{24}(x) = 1093 = \log_a x \cdot (4372)$$

$$\Rightarrow x = a^{1/4} \quad \dots \text{(i)}$$

$$\text{Also, } S_{12}(2x) = 265 = (\log_a 2x) \cdot (530)$$

$$\Rightarrow 2x = a^{1/2} \quad \dots \text{(ii)}$$

from (i) and (2)

$$a^{1/2} = 2a^{1/4}$$

$$\Rightarrow a^2 = 16a \Rightarrow a = 16$$

- 17.** If  $\frac{1}{16}, a, b$  are in GP and  $\frac{1}{a}, \frac{1}{b}, 6$  are in AP, then

$$72(a+b) =$$

**Ans.** 14

**Sol.**  $a^2 = \frac{b}{16}$  and  $\frac{2}{b} = \frac{1}{a} + 6$

Solving, we get  $a = \frac{1}{12}$  or  $a = \frac{-1}{4}$  [rejected]

$$\text{If } a = \frac{1}{12} \Rightarrow b = \frac{1}{9}$$

$$\therefore 72(a+b) = 72 \left( \frac{1}{12} + \frac{1}{9} \right) = 14$$

- 18.** The number of points lying in the interior of line segments AB, BC, CD, DA of rectangle ABCD are 5, 6, 7 and 9. If  $\alpha$  is the number of triangle made from the points by selecting each vertex from different line segments and  $\beta$  is the number of such quadrilaterals then the value of  $|\alpha - \beta|$  is

**Ans.** (2)

$$\text{Sol. } \alpha = {}^6C_1 {}^7C_1 {}^9C_1 + {}^5C_1 {}^7C_1 {}^9C_1 + {}^5C_1 {}^6C_1 {}^9C_1 + {}^5C_1 {}^6C_1 {}^7C_1 = 378 + 315 + 270 + 210 = 1173$$

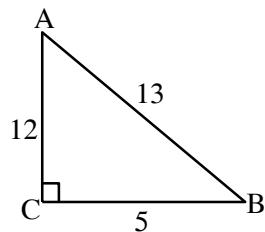
$$\beta = {}^5C_1 \cdot {}^6C_1 \cdot {}^7C_1 \cdot {}^9C_1 = 1890$$

$$\Rightarrow \beta - \alpha = 1890 - 1173 = 717$$

- 19.** Let a, b are sides of triangle where  $a = 5$ ,  $b = 12$  and area of triangle is 30, then value of  $2R + r$

**Ans.** (2)

**Sol.**       $\angle C = 90^\circ$



$$R = \frac{13}{2}, S = \frac{12+5+13}{2} = 15$$

$$r = \frac{\Delta}{S} = \frac{30}{15} = 2$$

$$2R + r = 13 + 2 = 15$$

- 20.** Find the interval in which  $f(x) = \log_e \left| \frac{x-1}{x+1} \right| - \frac{2}{x-1}$  is increasing

$$(1) (-\infty, -1) \cup [0, 1) \cup (1, \infty)$$

$$(2) (-\infty, -1) \cup [0, 1) \cup (1, \infty)$$

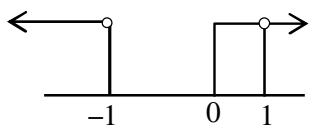
(3)  $(-1, -\infty)$

$$(4) (-\infty, -1) \cup (-1, 1)$$

**Ans.** (1)

$$\begin{aligned}
 \text{Sol. } f'(x) &= \frac{x+1}{x-1} \times \frac{x+1-(x-1)}{(x+1)^2} + \frac{2}{(x-1)^2} = \frac{2}{(x-1)(x+1)} + \frac{2}{(x-1)^2} \\
 &= \frac{2}{(x-1)} \left( \frac{1}{x+1} + \frac{1}{x-1} \right) = \frac{2}{(x-1)^2(x+1)} (2x) \\
 &= \frac{4x}{(x-1)^2(x+1)}
 \end{aligned}$$

for increasing,  $y'(x) \geq 0$



$$\Rightarrow \frac{x}{(x-1)^2(x+1)} \geq 0$$

$$\Rightarrow x \in (-\infty, -1) \cup [0, 1] \cup (1, \infty)$$



**Ans.** (7)

**Sol.** (a, b) R (c, d)  $\Rightarrow ad = bc$

$$\therefore (a, b) R (3, 4) \Rightarrow 4a = 3b \Rightarrow a = \frac{3}{4}b$$



**Ans.** (2)

$$\text{Sol.} \quad \vec{c} = \lambda(\vec{a} \times \vec{b})$$

$$\vec{c} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = \lambda[i(1-4) - j(1-2) + k(2-1)]$$

$$\vec{c} = \lambda(-3\hat{i} + \hat{j} + \hat{k})$$

$$\vec{c} \cdot (\hat{i} - \hat{j} + 3\hat{k}) = 8$$

$$\lambda(-3\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} - \hat{j} + 3\hat{k}) = 8$$

$$\lambda(-3 - 1 + 3) = 8$$

$$-\lambda = 8$$

$$\lambda = -8$$

$$\vec{c} = -8(\vec{a} \times \vec{b})$$

$$\vec{c} \cdot (\vec{a} \times \vec{b}) = -8 |\vec{a} \times \vec{b}|^2 = -8(\sqrt{9+1+1})^2 = -88$$

- 23.** Point  $(1, -1, 2)$  is the foot of perpendicular drawn from point  $(0, 3, 1)$  on the line  $\frac{x-a}{\ell} = \frac{y-2}{3} = \frac{z-b}{4}$ . find the shortest distance between this line and the line  $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{5}$ .

(1)  $\frac{61}{\sqrt{1314}}$

(2)  $\frac{71}{\sqrt{1314}}$

(3)  $\frac{91}{\sqrt{1314}}$

(4)  $\frac{31}{\sqrt{1314}}$

**Ans.** (1)

**Sol.** Let  $A(0, 3, 1)$  and  $B(1, -1, 2)$

d·r's of  $AB$  are  $1, -4, 1$

$$\ell(1) + 3(-4) + 4(1) = 0 \Rightarrow \ell = 8$$

$$B \text{ line on } L_1 \Rightarrow \frac{1-a}{8} = -1 = \frac{2-b}{4}$$

$$\Rightarrow a = 9, b = 6$$

$$\Rightarrow L_1 \text{ is } \frac{x-9}{8} = \frac{y-2}{3} = \frac{z-6}{4}$$

$$L_2 \text{ is } \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{5}$$

$$\begin{vmatrix} 8 & 0 & 3 \\ 8 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$\text{Shortest distance} = \frac{61}{\left| \hat{i} - 28\hat{j} + 23\hat{k} \right|} = \frac{61}{\sqrt{1+784+529}} = \frac{61}{\sqrt{1314}}$$