

JEE
MAIN
2020

NARAYANA'S SENSATIONAL SUCCESS ACROSS INDIA

NARAYANA IIT-JEE (MAIN) 2020 ALL INDIA TOP RANKS IN OPEN CATEGORY



SWAYAM CHUBE
HT No. MR16303069

BELOW
10 **21** RANKS

*All Cat



**HARSHVARDHAN
AGARWAL**
HT No. HR09300789

BELOW
100 **126** RANKS

*All Cat



SHIVA KRISHNA
HT No. TL01330751

TOTAL
QUALIFIED
FOR JEE-ADV. **16292**

ADMISSIONS OPEN (2020-21)

OUR REGULAR CLASSROOM PROGRAMME

One Year Classroom Program
JEE/NEET-2021
(for students moving from XI to XII)

Two Year Classroom Program
JEE/NEET-2022
(for students moving from X to XI)

Three Year Integrated Classroom Program
JEE/NEET-2023
(for students moving from IX to X)

Four Year Integrated Classroom Program
JEE/NEET-2024
(for students moving from VIII to IX)

FOUNDATION PROGRAMMES
For NTSE, NSEJS, JSTSE,
Olympiads & School/Board Exams
(for students moving to
Class VI, VII, VIII, IX & X)

APEX BATCH
Two years school Integrated
Classroom Program - 2022
For JEE Main & Advance / NEET (for XI Studying Students)
Course Feature - Complete Coverage of CBSE - Regular Classes - Weekly Test & Regular Analysis - Lab Facility
- Motivation & Counseling - Competitive Exam Prep - Ample time for self study

Online Classes for IIT/NEET/Foundation/Olympiads

- Access Recording of Past Classes on n-Learn App
- Online Parent Teacher Meeting
- Personalized Extra Classes & Live Doubt Solving
- Hybrid/Customized Classroom model
- Video Solution of Weekly/Fortnightly Test
- Printed Study Material will be sent by us
- n-Learn App
- Counselling Motivational sessions
- Affordable Fee
- Doubt Classes / Practice Classes
- Provision to Convert from online to regular classroom programme
- Once Classes resume by just paying nominal fee

Online Test

- Micro & Macro Analysis
- Relative performance (All India Ranking)
- Question wise Analysis
- Unlimited Practice Test
- Grand Test

NARAYANA
Digital
Classes
STUDY ONLINE FROM HOME

For Class
7th to 12th+



JEE-MAIN-2021

FEBRUARY ATTEMPT

24.02.21_SHIFT-II

MATHEMATICS

MATHEMATICS

1. Find the value of ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + \dots + {}^nC_2) = ?$

(1) $\frac{n(n+1)(2n-1)}{6}$ (2) $\frac{n(n+1)(2n+1)}{6}$
 (3) $\frac{(n-1)n(n+1)}{6}$ (4) $\frac{n(n+1)}{2}$

Ans. (2)

Sol. $S = {}^2C_2 + {}^3C_2 + \dots + {}^nC_2 = {}^{n+1}C_3$

$\therefore {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$

$= \frac{(n+1)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$

$= \frac{(n+2)(n+1)n}{6} + \frac{(n+1)(n)(n-1)}{6} = \frac{n(n+1)}{6} (2n+1)$

2. If A and B are subset s of $X = \{1,2,3,4,5\}$ then find the probability such that $n(A \cap B) = 2$.

(1) $\frac{65}{2^7}$ (2) $\frac{65}{2^9}$ (3) $\frac{35}{2^9}$ (4) $\frac{135}{2^9}$

Ans. (4)

Sol. Required probability

$= \frac{{}^5C_2 \times 3^3}{4^5}$

$= \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$

3. Given $f(0) = 1$, $f(2) = e^2$ also $f'(x) = f'(2-x)$, then the value of $\int_0^2 f(x)dx$ is

(1) $1 - e^2$ (2) $1 + e^2$ (3) e (4) e^2

Ans. (2)

Sol. $f'(x) = f'(2-x)$

On integrating both side $f(x) = -f(2-x) + c$

put $x = 0$

$f(0) + f(2) = c \Rightarrow c = 1 + e^2$

$\Rightarrow f(x) + f(2-x) = 1 + e^2 \dots \dots (i)$

$I = \int_0^2 f(x) dx = \int_0^1 \{f(x) + f(2-x)\} dx = (1 + e^2)$

4. A curve $y = f(x)$ passing through the point $(1,2)$ satisfies the differential equation $x \frac{dy}{dx} + y = bx^4$

such that $\int_1^2 f(y)dy = \frac{62}{5}$. The value of b is

- (1) 10 (2) 11 (3) $\frac{32}{5}$ (4) $\frac{62}{5}$

Ans. (1)

Sol. $\frac{dy}{dx} + \frac{y}{x} = bx^3$

I.F. = $e^{\int \frac{dx}{x}} = x$

$\therefore yx = \int bx^4 dx = \frac{bx^5}{5} + C$

Passes through $(1,2)$, we get

$2 = \frac{b}{5} + C$..(i)

Also, $\int_1^2 \left(\frac{bx^4}{5} + \frac{C}{x} \right) dx = \frac{62}{5}$

$\Rightarrow \frac{b}{25} \times 32 + C \ln 2 - \frac{b}{25} = \frac{62}{5}$

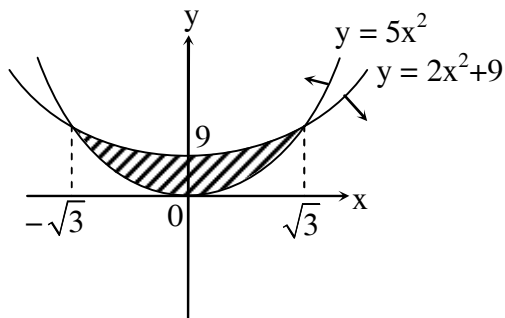
$\Rightarrow C = 0$ & $b = 10$

5. The area of the region defined by $5x^2 \leq y \leq 2x^2 + 9$ is

- (1) $6\sqrt{3}$ (2) $12\sqrt{3}$ (3) $18\sqrt{3}$ (4) $9\sqrt{3}$

Ans. (2)

Sol.



Required area

$= 2 \int_0^{\sqrt{3}} (2x^2 + 9 - 5x^2) dx$

$= 2 \int_0^{\sqrt{3}} (9 - 3x^2) dx$

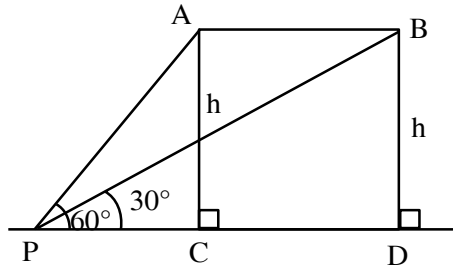
$= 2 \left[9x - x^3 \right]_0^{\sqrt{3}} = 12\sqrt{3}$

6. A aeroplane is flying horizontally with speed of 432 km/hr at height h meter from ground. Its angle of elevation from a point on ground is 60° . After 20 sec its angle of elevation from same point is 30° then the height ' h ' is equal to

- (1) $1200\sqrt{3}$ (2) $600\sqrt{3}$ (3) $1800\sqrt{3}$ (4) $1000\sqrt{3}$

Ans. (1)

Sol.



$$v = 432 \times \frac{1000}{60 \times 60} \text{ m/sec} = 120 \text{ m/sec}$$

$$\text{Distance } AB = v \times 20 = 2400 \text{ meter}$$

In ΔPAC

$$\tan 60^\circ = \frac{h}{PC} \Rightarrow PC = \frac{h}{\sqrt{3}}$$

In ΔPBD

$$\tan 30^\circ = \frac{h}{PD} \Rightarrow PD = \sqrt{3}h$$

$$PD = PC + CD$$

$$\sqrt{3}h = \frac{h}{\sqrt{3}} + 2400 \Rightarrow \frac{2h}{\sqrt{3}} = 2400$$

$$h = 1200\sqrt{3} \text{ meter}$$

7. A curve $y = ax^2 + bx + c$ passing through the point (1, 2) has slope at origin equal to 1. then ordered triplet (a, b, c) may be

- (1) (1, 1, 0) (2) $\left(\frac{1}{2}, 1, 0\right)$ (3) $\left(-\frac{1}{2}, 1, 1\right)$ (4) (2, -1, 0)

Ans. (1)

Sol. $2 = a + b + c$ (i)

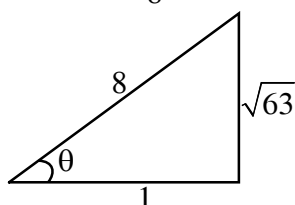
$$\frac{dy}{dx} = 2ax + b \Rightarrow \frac{dy}{dx}\bigg|_{(0,0)} = 1$$

$$\Rightarrow b = 1 \Rightarrow a + c = 1$$

8. The value of $\tan \left(\frac{1}{4} \sin^{-1} \frac{\sqrt{63}}{8} \right)$ is
- (1) $\frac{1}{\sqrt{7}}$ (2) $\frac{1}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{3}}$ (4) none of these

Ans. (1)

Sol. Let $\sin^{-1} \frac{\sqrt{63}}{8} = \theta \Rightarrow \sin \theta = \frac{\sqrt{63}}{8}$



$$\tan \left(\frac{1}{4} \sin^{-1} \frac{\sqrt{63}}{8} \right) = \tan \left(\frac{\theta}{4} \right) = \frac{1 - \cos \frac{\theta}{2}}{\sin \frac{\theta}{2}} = \frac{1 - \sqrt{\frac{1 + \cos \theta}{2}}}{\sqrt{\frac{1 - \cos \theta}{2}}} = \frac{1 - \frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{1}{\sqrt{7}}$$

9. The value of $\int_1^3 [x^2 - 2x - 2] dx$ ([.] denotes greatest integers function)

- (1) -4 (2) -5 (3) $-1 - \sqrt{2} - \sqrt{3}$ (4) $1 - \sqrt{2} - \sqrt{3}$

Ans. (3)

Sol. $I = \int_1^3 -3 dx + \int_1^3 [(x-1)^2] dx$ $x - 1 = t; dx = dt$

$$I = (-6) + \int_0^2 [t^2] dt$$

$$I = -6 + \int_0^1 0 dt + \int_1^{\sqrt{2}} 1 dt + \int_{\sqrt{2}}^{\sqrt{3}} 2 dt + \int_{\sqrt{3}}^2 3 dt$$

$$I = -6 + (\sqrt{2} - 1) + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$$

$$I = -1 - \sqrt{2} - \sqrt{3}$$

10. Which of the following conic has tangent ' $x + \sqrt{3}y - 2\sqrt{3}$ ' at point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2} \right)$?

- (1) $x^2 + 9y^2 = 9$ (2) $y^2 = \frac{x}{6\sqrt{3}}$ (3) $x^2 - 9y^2 = 10$ (4) $x^2 = \frac{y}{6\sqrt{3}}$

Ans. (1)

Sol. tangent to $x^2 + 9y^2 = a$ at point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2} \right)$ is $x \left(\frac{3\sqrt{3}}{2} \right) + 9y \left(\frac{1}{2} \right) = 9$

\Rightarrow option (1) is true

11. The negation of the statement $\sim p \wedge (p \vee q)$ is

- (1) $p \wedge \sim q$ (2) $p \vee \sim q$ (3) $\sim p \wedge q$ (4) $\sim p \vee \sim q$

Ans. (2)

Sol. $\sim(\sim p \wedge (p \vee q))$

$$= \sim((\sim p \wedge p) \vee (\sim p \wedge q))$$

$$= \sim(\sim p \wedge q) = p \vee \sim q$$

12. Equation of plane passing through (1, 0, 2) and line of intersection of planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} - 2\hat{j}) = -2$ is

- (1) $\vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$ (2) $\vec{r} \cdot (3\hat{i} + 10\hat{j} + 3\hat{k}) = 7$
 (3) $\vec{r} \cdot (\hat{i} + \hat{j} - 3\hat{k}) = 4$ (4) $\vec{r} \cdot (\hat{i} + 4\hat{j} - \hat{k}) = -7$

Ans. (1)

Sol. Plane passing through intersection of plane is

$$\{\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = -1\} + \lambda \{\vec{r} \cdot (\hat{i} - 2\hat{j}) + 2\} = 0$$

Passes through $\hat{i} + 2\hat{k}$, we get

$$(3 - 1) + \lambda(\lambda + 2) = 0 \quad \Rightarrow \quad \lambda = -\frac{2}{3}$$

$$\text{Hence, equation of plane is } 3\{\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1\} - 2\{\vec{r} \cdot (\hat{i} - 2\hat{j}) + 2\} = 0$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + 7\hat{j} + 3\hat{k}) = 7$$

13. A is 3×3 square matrix and B is 3×3 skew symmetric matrix and X is a 3×1 matrix, then equation $(A^2B^2 - B^2A^2)X = 0$ (Where O is a null matrix) has/have

- (1) Infinite solution (2) No Solution
 (3) Exactly one solution (4) Exactly two solution

Ans. (1)

Sol. $A^T = A, B^T = -B$

$$\text{Let } A^2B^2 - B^2A^2 = P$$

$$\begin{aligned} P^T &= (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T \\ &= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T \\ &= B^2A^2 - A^2B^2 \end{aligned}$$

$$\Rightarrow P \text{ is skew-symmetric matrix}$$

$$\Rightarrow |P| = 0$$

Hence $PX = 0$ have infinite solution

14. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, $f(0) = 1$ and $f'(0) = 2$ then $f(1)$ belongs to interval

- (1) $[6, 9]$ (2) $[9, 12]$ (3) $[8, 10]$ (4) $[5, 7]$

Ans. (1)

Sol. Given $f(x) f''(x) - (f'(x))^2 = 0$

Let $h(x) = \frac{f(x)}{f'(x)}$

$\Rightarrow h'(x) = 0 \quad \Rightarrow h(x) = k$

$\Rightarrow \frac{f(x)}{f'(x)} = k \quad \Rightarrow f'(x) = \frac{f(x)}{k}$

$\Rightarrow f(x) = k f'(0) \quad \Rightarrow 1 = k(2) \Rightarrow k = \frac{1}{2}$

New $f(x) = \frac{1}{2} f'(x) \Rightarrow \int 2dx = \int \frac{f'(x)}{f(x)} dx$

$\Rightarrow 2x = \ln |f(x)| + C$

As $f(0) = 1 \Rightarrow C = 0$

$\Rightarrow 2x = \ln |f(x)| \Rightarrow f(x) = \pm e^{2x}$

As $f(0) = 1 \Rightarrow f(x) = e^{2x} \Rightarrow f(1) = e^2$

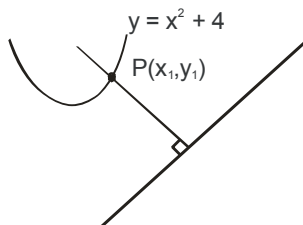
15. Find a point on the curve $y = x^2 + 4$ which is at shortest distance from the line $y = 4x - 1$.

- (1) (2,8) (2) (1,5) (3) (3,13) (4) (-1,5)

Ans. (1)

Sol. $\left. \frac{dy}{dx} \right|_p = 4$

$\therefore 2x_1 = 4$



$\Rightarrow x_1 = 2$

\therefore Point will be (2,8)

16. Let $f(x) = \begin{cases} -55x & ; \quad x < -5 \\ 2x^3 - 3x^2 - 120x & ; \quad -5 \leq x < 4 \\ 2x^3 - 3x^2 - 36x + 10 & ; \quad x \geq 4 \end{cases}$

Then interval in which $f(x)$ is monotonically increasing is

- (1) $(-5, -4) \cup (4, \infty)$ (2) $(-\infty, -4) \cup (5, \infty)$
 (3) $(-5, 4) \cup (5, \infty)$ (4) $(-5, -4) \cup (3, \infty)$

Ans. (1)

Sol. $f'(x) = \begin{cases} -55 & ; \quad x < -5 \\ 6(x^2 - x - 20) & ; \quad -5 < x < 4 \\ 6(x^2 - x - 6) & ; \quad x > 4 \end{cases}$

$$f'(x) = \begin{cases} -55 & ; \quad x < -5 \\ 6(x-5)(x+4) & ; \quad -5 < x < 4 \\ 6(x-3)(x+2) & ; \quad x > 4 \end{cases}$$

Hence, $f(x)$ is monotonically increasing is $(-5, -4) \cup (4, \infty)$

17. If a, b, c are in A.P. & centroid of the triangle with vertices $(a, c), (a, b), (2, b)$ is $\left(\frac{10}{3}, \frac{7}{3}\right)$ and

α, β are roots of the equation $ax^2 + bx + 1 = 0$, then $\alpha^2 + \beta^2 - \alpha\beta$ equals

- (1) $-\frac{71}{256}$ (2) $\frac{71}{256}$ (3) $\frac{69}{256}$ (4) $-\frac{69}{256}$

Ans. (1)

Sol. $2b = a + c$

$$\frac{2a+2}{3} = \frac{10}{3} \text{ and } \frac{2b+c}{3} = \frac{7}{3}$$

$$\Rightarrow \left. \begin{array}{l} a = 4 \\ 2b + c = 7 \\ 2b - c = 4 \end{array} \right\} \text{ solving,}$$

$$b = \frac{11}{4} \quad c = \frac{3}{2}$$

$$\therefore \text{ Quadratic Equation is } 4x^2 + \frac{11}{4}x + 1 = 0$$

$$\therefore \text{ The value of } (\alpha + \beta)^2 - 3\alpha\beta = \frac{121}{256} - \frac{3}{4} = -\frac{71}{256}$$

- 18.** Given $a + \alpha = 1$, $b + \beta = 2$ and $\alpha f(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ then value of $\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}}$

Ans. 2

Sol. $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ (i)

$$x \rightarrow \frac{1}{x}$$

$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x$ (ii)

(i) + (ii)

$$(a + \alpha) \left[f(x) + f\left(\frac{1}{x}\right) \right] = \left(x + \frac{1}{x} \right) (b + \beta)$$

$$\frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{2}{1} = 2$$

- 19.** Find the maximum value of 'k' for which the maximum value of variance of 10 elements is 10 in which 9 values are 1 and one value of is k. (Where k is integer)

Ans. 11

Sol. $\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n} \right)^2$

$$\sigma^2 = \frac{(9 + k^2)}{10} - \left(\frac{9 + k}{10} \right)^2 < 10$$

$$(90 + k^2)10 - (81 + k^2 + 8k) < 1000$$

$$90 + 10k^2 - k^2 - 18k - 81 < 1000$$

$$9k^2 - 18k + 9 < 1000$$

$$(k - 1)^2 < \frac{1000}{9} \Rightarrow k - 1 < \frac{10\sqrt{10}}{3}$$

$$k < \frac{10\sqrt{10}}{3} + 1$$

Maximum integral value of k = 11

- 20.** Distance of P(x, y) from (5,0) is thrice as distance of P(x, y) from (-5,0). If locus of P is circle with radius 'r' then find the value of $4r^2$.

Ans. 56.25

Sol. Internal point which divide (5,0) & (-5,0) in the ratio 3 : 1 is $\left(\frac{-5}{2}, 0\right)$ External point which divide (5,0) & (-5,0) in the ratio 3 : 1 is (-10,0)

$$2r = \left(\frac{-5}{2} + 10\right) = \frac{15}{2} = 7.5$$

$$(2r)^2 = 56.25$$

- 21.** Four numbers whose sum is $\frac{65}{12}$ are in G.P. Sum of their reciprocals is $\frac{65}{18}$ and product of first three of them is 1. If third term is α then find value of 2α .

Ans. 3

Sol. a, ar, ar^2 , ar^3

$$a + ar + ar^2 + ar^3 = \frac{65}{12} \quad \text{..... (i)}$$

$$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$$

$$\frac{1}{a} \left(\frac{r^3 + r^2 + r + 1}{r^3} \right) = \frac{65}{18} \quad \text{..... (ii)}$$

$$\frac{(i)}{(ii)}, a^2 r^3 = \frac{18}{12} = \frac{3}{2}$$

$$a^3 r^3 = 1 \Rightarrow a \left(\frac{3}{2} \right) = 1 \Rightarrow a = \frac{2}{3}$$

$$\frac{4}{9} r^3 = \frac{3}{2} \Rightarrow r^3 = \frac{3^3}{2^3} \Rightarrow r = \frac{3}{2}$$

$$\alpha = ar^2 = \frac{2}{3} \cdot \left(\frac{3}{2} \right)^2 = \frac{3}{2}$$

$$2\alpha = 3$$

- 22.** There are 10 students S_1, S_2, \dots, S_{10} . Find the number of ways to form 3 groups G_1, G_2, G_3 such that all groups has at least 1 member and group G_3 has at most 3 members

Ans. 26650

Sol.

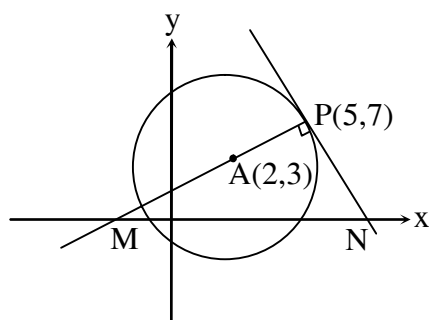
A	B	C
1	8	1
2	7	1
\vdots	\vdots	\vdots
6	1	3

$$\begin{aligned} \text{Ways to distribute in groups} &= {}^{10}C_1({}^9C_1 + \dots + {}^9C_8) + {}^{10}C_2({}^8C_1 + \dots + {}^8C_7) + {}^{10}C_3({}^7C_1 + \dots + {}^7C_6) \\ &= 10(510) + 45(254) + 120(126) \\ &= 26650 \end{aligned}$$

- 23.** At point $P(5, 7)$ on circle $(x - 2)^2 + (y - 3)^2 = 25$ a tangent and a normal is drawn. The area of triangle formed by this tangent normal with x axis is λ then 24λ is

Ans. 1225

Sol.



equation of normal at P

$$(y - 7) = \left(\frac{7-3}{5-2} \right) (x - 5)$$

$$3y - 21 = 4x - 20$$

$$\Rightarrow 4x - 3y + 1 = 0 \quad \dots\dots\dots (i)$$

$$\Rightarrow M\left(-\frac{1}{4}, 0\right)$$

equation of tangent at P

$$(y - 7) = -\frac{3}{4} (x - 5)$$

$$4y - 28 = -3x + 15$$

$$\Rightarrow 3x + 4y = 43 \quad \dots\dots\dots (ii)$$

$$\Rightarrow N\left(\frac{43}{3}, 0\right)$$

$$\text{hence ar}(\triangle PMN) = \frac{1}{2} \times MN \times 7$$

$$l = \frac{1}{2} \times \frac{175}{12} \times 7$$

$$\Rightarrow 24\lambda = 1225$$