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MATHEMATICS

If $z^2 + \alpha z + \beta = 0$ has one root 1 - 2i, where $\alpha, \beta, \in R$ find the value of $\alpha - \beta$ 1. (2) 10(3) - 7(4)7(1) 12Ans. (3) If the root is 1 - 2i, the other roots is 1 + 2iSol. Sum = 2, Product = 5 \therefore quadratic equation $z^2 - 2z + 5 = 0$ $\Rightarrow \alpha = -2, \beta = 5$ $\alpha - \beta = -2 - 5 = -7$ Find the value of cosec $\left(2 \cot^{-1}(5) + \cos^{-1}\left(\frac{4}{5}\right)\right)$ 2. (2) $\frac{56}{65}$ (3) $\frac{63}{65}$ $(1) \frac{65}{56}$ $(4) \frac{65}{63}$ Ans. (1) $= \operatorname{cosec}\left(\operatorname{tan}^{-1}\left(\frac{1}{5}\right) + \operatorname{tan}^{-1}\left(\frac{1}{5}\right) + \operatorname{tan}^{-1}\left(\frac{3}{4}\right)\right)$ Sol. $= \operatorname{cosec}\left(\operatorname{tan}^{-1}\left(\frac{5}{12}\right) + \operatorname{tan}^{-1}\left(\frac{3}{4}\right)\right)$ 65 56 33 $= \operatorname{cosec}\left(\operatorname{tan}^{-1}\left(\frac{56}{33}\right)\right)$ $=\frac{65}{56}$

3. If $a_n = \alpha^n - \beta^n$ and α , β are the roots of the equation $x^2 - 6x - 2 = 0$ find the value of $\frac{a_{10} - 2a_8}{3a_9}$ (1) 2 (2) -2 (3) 3 (4) -3





Sol.
$$E = \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{3(\alpha^9 - \beta^9)} = \frac{6(\alpha^9 - \beta^9)}{3(\alpha^9 - \beta^9)} = 2$$

4. If $f(x) = \frac{5^x}{5 + 5^x} \text{ find } f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + \dots + f\left(\frac{39}{20}\right)$
(1) 20 (2) $\frac{29}{2}$ (3) $\frac{19}{2}$ (4) $\left(\frac{39}{2}\right)$
Ans. (4)
Sol. $f(x) = \frac{5^{x}}{5 + 5^x}$
 $f(2 - x) = \frac{5^{2-x}}{5 + 5^{2-x}}$
 $= \frac{25}{5.5^x + 25} = \frac{5}{5^x + 5}$
 $f(x) + f(2 - x) = 1$
Now $\left(f\left(\frac{1}{20}\right) + f\left(\frac{39}{20}\right)\right) + \left(f\left(\frac{2}{20}\right) + f\left(\frac{38}{20}\right)\right) + \dots \left(f\left(\frac{19}{12}\right) + f\left(\frac{21}{20}\right)\right) + f\left(\frac{20}{20}\right)$
 $= 1 \times 19 + \frac{1}{2} = \frac{39}{2}$

5.
$$\lim_{n \to \infty} \left(\frac{1}{n} + \frac{n}{(n+1)} + \frac{n}{(n+2)^2} + \dots + \frac{n}{(2n-1)^2} \right) \text{ is equal to}$$
(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) 1 (4) $\frac{2}{3}$

Sol.
$$1 = \lim_{n \to \infty} \sum_{r=0}^{n-1} \frac{n}{(n+r)^2}$$

 $\therefore L = \lim_{n \to \infty} \frac{1}{n} \sum_{r=0}^{n-1} \frac{1}{\left(\frac{r}{n}\right)^2 + 2\left(\frac{r}{n}\right) + 1}$
 $\therefore L = \int_0^1 \frac{dx}{(x+1)^2} = \frac{-1}{x+1} \Big|_0^1$
 $L = -\frac{1}{2} + 1 = \frac{1}{2}$





| 6. | The system $2x + 3y + 2z = 1$, $4x + 6y + 2z = 1$, $-x + y + 2z = 3$ has | | | | | | | | |
|------|--|--|-----------------------|-------------------|--|--|--|--|--|
| | (1) unique solution | (2) no solution | (3) infinite solution | (4) none of these | | | | | |
| Ans. | (1) | | | | | | | | |
| | 2 +3 2 | | | | | | | | |
| Sol. | $D = \begin{vmatrix} 4 & 6 & 2 \end{vmatrix} = 2(10) - 3(10) + 2(10) \neq 0$ | | | | | | | | |
| | -1 1 2 | | | | | | | | |
| | so unique solution | | | | | | | | |
| | - | | | | | | | | |
| 7. | If A is a 3 \times 3 matrix and $ A = 4$ and R_i denote the i th row of A, if matrix B is formed using | | | | | | | | |
| | $R_2 \rightarrow 2R_2 + 5R_3$ in matrix 2A then find B . | | | | | | | | |
| | (1) 64 | (2) 80 | (3) 128 | (4) 16 | | | | | |
| Ans. | (1) | | | | | | | | |
| Sol. | A = 4 | | | | | | | | |
| | $ 2A = 2^3 A = 8 \times 4$ | | | | | | | | |
| | Now $R_2 \rightarrow 2R_2 + 5R_2$ | R ₃ | | | | | | | |
| | $ \mathbf{B} = 2 \times 32$ | | | | | | | | |
| | = 64 | | | | | | | | |
| | | | | | | | | | |
| 8. | $\lim_{x \to 0} \frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)} = t$ | Dif limit exist then find | 2(a + b) ? | | | | | | |
| | (1)-1 | (2) –7 | (3) 1 | (4) 7 | | | | | |
| Ans. | (4) | | | | | | | | |
| Sol. | $\lim_{x\to 0}\frac{ax-(e^{4x}-1)}{ax.4x}$ | | | | | | | | |
| | Apply L' Hospital | | | | | | | | |
| | $a - (e^{4x}).4$ | a-4 | | | | | | | |
| | $\lim_{x\to 0} \frac{1}{8ax}$ | $\left(\frac{u}{0}\right)$ form) | | | | | | | |
| | 4 limit exist $a = 4$ | | | | | | | | |
| | $\lim_{x \to 0} \frac{4 - 4e^{4x}}{32x} = \lim_{x \to 0} \frac{1}{2}$ | $\frac{-\mathrm{e}^{4\mathrm{x}}}{8\mathrm{x}} = \frac{-1}{2}$ | | | | | | | |
| | $a = 4$, $b = \frac{-1}{2}$ | | | | | | | | |
| | $2(a+b) = 2\left(4-\frac{1}{2}\right)$ |)=7 | | | | | | | |
| | | | | | | | | | |
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| | | | | | | | | | |





9. A is the set of all four digit natural numbers which has exactly one digit '7'. Find the probability of choosing a number from the set A so that it leaves the remainder 2 when divided by 5.

(1)
$$\frac{97}{297}$$
 (2) $\frac{91}{297}$ (3) $\frac{37}{297}$ (4) None of these

10. Minimum value of
$$y = a^{a^{x}} + \frac{a}{a^{a^{x}}}$$
 is $(a > 0, a, x \in \mathbb{R})$
(1) $2\sqrt{a}$ (2) $\sqrt{2}a$ (3) $2\sqrt{2}a$ (4) $2\sqrt{2a}$

Ans. (1)

Sol. A.M
$$\ge$$
 G.M $\Rightarrow \frac{a^{a^x} + \frac{a}{a^{a^x}}}{2} \ge \left(a^{a^x} \times \frac{a}{a^{a^x}}\right)^{1/2}$

$$a^{a^x} + \frac{a}{a^{a^x}} \ge 2\sqrt{a}$$

- \therefore Minimum value = $2\sqrt{a}$
- 11. A hyperbola passes through the focii of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and the product of their eccentricities is 1. Equation of hyperbola is

(1)
$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$
 (2) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (3) $\frac{x^2}{9} - \frac{y^2}{5} = 1$ (4) $\frac{x^2}{5} - \frac{y^2}{9} = 1$

Ans. (2)





Sol.
$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

 $a = 5, b = 4$
 $e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$
focii : (3, 0), (-3, 0)
let equation of hyperbola is $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$
satisfy (±3, 0) $\Rightarrow \frac{9}{A^2} = 1 \Rightarrow A^2 = 9$
eccentricity of hyperbola $= \frac{1}{\text{eccentricity of ellipse}} = \frac{5}{3}$
 $\Rightarrow \frac{5}{3} = \sqrt{1 + \frac{B^2}{9}} \Rightarrow 1 + \frac{B^2}{9} = \frac{25}{9}$
 $\Rightarrow B^2 = 16$
equation of hyperbola is
 $\frac{x^2}{9} - \frac{y^2}{16} = 1$

12. The contrapositive of the statement :

"If you work hard then you will earn" is :

- (1) If you don't work hard you will earn
- (2) If you will not earn then you not work hard .
- (3) you will work hard and you will not earn.
- (4) Either you will work hard or you will earn

Ans. (2)

- **Sol.** p : you work ward
 - q : you will earn

given
$$(p \rightarrow q)$$

contrapositive of $(p \rightarrow q) = \sim q \rightarrow \sim p$

13. Minimum distance between the curves $x^2 = 2y$ and y = x - 1 is :

(1)
$$\frac{1}{2\sqrt{2}}$$
 (2) $\frac{1}{3\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$





Sol.
$$\frac{dy}{dx}\Big|_{P} = 1$$

$$P(x_1, y_1)$$

$$\therefore x_1 = 1$$

$$P(x_1, y_2)$$

$$P($$

14.
$$I_n = \int_{\pi/4}^{\pi/2} \cot^n x \, dx$$
, then $I_2 + I_4$, $I_3 + I_5$, $I_4 + I_6$ are in
(1) A.P. (2) G.P. (3) H.P. (4) none of these
Ans. (3)

Sol.
$$I_n = \int_{\pi/4}^{\pi/2} (\cot x)^n dx$$

 $I_n + I_{n+2} = \int_{\pi/4}^{\pi/2} ((\cot x)^n + (\cot x)^{n+2}) dx = \int_{\pi/4}^{\pi/2} (\cot x)^n \csc^2 x dx$
 $\cot x = t$
 $= -\int_{1}^{0} t^n dx = \int_{0}^{1} t^n dx = \frac{t^{n+1}}{n+1} \Big|_{0}^{1} = \frac{1}{n+1}$
 $\therefore I_n + I_{n+2} = \frac{1}{n+1}$
 $\therefore I_2 + I_4 = \frac{1}{3}$
 $I_3 + I_5 = \frac{1}{4}$
 $I_4 + I_6 = \frac{1}{5}$
 $\Rightarrow I_2 + I_4, I_3 + I_5, I_4 + I_6 \text{ are in H.P.}$





15. If $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, 0 < x, $y < \pi$, find the value of $\sin x + \cos y$.

- (1) $\frac{1}{2}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1+\sqrt{3}}{2}$ (4) $\frac{1-\sqrt{3}}{2}$
- **Ans.** (3)
- **Sol.** $x = y = \frac{\pi}{3}$ satisfy the equation

$$\therefore \sin x + \cos y = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3} + 1}{2}$$

16. Find the projection of \overrightarrow{OP} on the plane passes through the point A(2, 1, 3), B(3, 2, 1), C(2, 4, 2); where P is the point (2, -1, 0)

(1)
$$\sqrt{\frac{44}{35}}$$
 (2) $\sqrt{\frac{47}{35}}$ (3) $\sqrt{\frac{33}{47}}$ (4) $\sqrt{\frac{41}{36}}$

Sol.
$$n = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ -1 & 2 & 1 \end{vmatrix} = 5\hat{i} + \hat{j} + 3\hat{k}$$

$$\int_{-1}^{n} \frac{1}{2 & 1} = 5\hat{i} + \hat{j} + 3\hat{k}$$

$$\int_{-1}^{n} \frac{1}{2 & 1} \frac{1}{2 & 1} = 5\hat{i} + \hat{j} + 3\hat{k}$$

$$\int_{-1}^{n} \frac{1}{2 & 1} \frac{1}{2 & 1} = \frac{1}{2 & 1} \hat{k}$$

$$\therefore \text{ Required plane is : } 5(x - 2) + (y - 1) + 3(z - 3) = 0$$

$$i.e. 5x + y + 3z = 20$$

$$|\overline{OP}| = \sqrt{4 + 1 + 0} = \sqrt{5}$$

$$|\overline{OP}| = 2\hat{i} - \hat{j}$$

$$\int_{-1}^{1} \frac{10 - 1}{\sqrt{5} \sqrt{25 + 1 + 9}} = \frac{9}{5\sqrt{7}}$$

$$\therefore \text{ Projection } = \sqrt{5} \times \cos \theta = \sqrt{5} \times \frac{1}{5} \sqrt{\frac{44}{7}} = \sqrt{\frac{44}{35}}$$





17. The line x + y = 1 cuts the ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$ in points P & Q. Find the angle subtended by



 $\sqrt{2},0$

Sol.

Ans.



angle between these line is $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

- 18. If a curve y(x) satisfies the differential equation $(2xy^2 y) dx + xdy = 0$ and it passes through the point of intersection of the line x + y = 4 and 2x 3y = -2, then find the value of |y(1)|.
- **Ans.** 0.5

Sol.
$$(2xy^2 - y) dx = -x dy$$

 $x \frac{dy}{dx} = y - 2xy^2$
 $\frac{dy}{dx} = \frac{1}{x} \cdot y - 2xy^2$
 $\frac{dy}{dx} - \frac{1}{x} \cdot y = -2xy^2$
 $y^{-2} \frac{dy}{dx} - \frac{1}{x} y^{-1} = -2$
 $y^{-1} = t \implies -y^{-2} \frac{dy}{dx} = \frac{dt}{dx}$





$$-\frac{dt}{dx} - \frac{1}{x} \quad t = -2$$

$$\Rightarrow \frac{dt}{dx} + \frac{1}{x} \quad t = 2 \quad \text{I.F.} = e^{cnx} = x$$

$$tx = 2 \times \frac{x^2}{2} + c \qquad \Rightarrow \frac{x}{y} = x^2 + c$$

It passes through P(2, 2)

$$\therefore \quad c = -3$$

$$\therefore \quad \frac{x}{y} = x^2 - 3$$

If $x = 1, \ \frac{1}{y} = -2 \qquad \Rightarrow y = -\frac{1}{2}$

$$\therefore \quad |y(1)| = \frac{1}{2} = 0.5$$

- 19. Let the curve $(x 3)^2 + y^2 = 9$ and $y^2 = 4x$ have common tangent touches the curves at (a, b) and (c, d) in Ist quadrant, then find 2(a + c)
- Ans. 9
- **Sol.** Equation of tangent of A



For point B foot of perpendicular from c to tangent

$$\frac{x-3}{1} = \frac{y-0}{-\sqrt{3}} = -\frac{(3-0+3)}{4} \implies x = \frac{3}{2}$$
$$c = \frac{3}{2} \text{ and } a = 3$$
$$2(a+c) = 9$$





If 'x' is a number divided by '4' leaves the remainder '3' then find the remainder if $(2020 + x)^{2022}$ is 20. divided by '8'.

Ans. 1

Sol.

 $x=4k+3 \ ; k \in W$ $\therefore (2020 + 4k + 3)^{2022} = (8\lambda + 1)^{1011}$ $\therefore (8\lambda + 1)^{1011} = {}^{1011}C_0 + \underbrace{{}^{1011}C_1(8\lambda) + \dots}_{\text{multiple of 8.}}$

 \therefore Remainder on dividing by 8 is 1

21. If
$$A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$$
 and $AA^{T} = I$ find $\alpha^{4} + \beta^{4}$

Ans. 1

Sol.

$$AA^{1} = I$$

$$\Rightarrow \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix} \begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+\alpha^{2} & \alpha-\alpha\beta \\ \alpha-\alpha\beta & \alpha^{2}+\beta^{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 1+\alpha^{2} = 1 \Rightarrow \alpha = 0$$

$$\alpha^{2}+\beta^{2} = 1 \Rightarrow \beta^{2} = 1$$

$$\therefore \alpha^{4}+\beta^{4} = 0+1 = 1$$

Find number of all two digit numbers 'n' such that $3^n + 7^n$ is divisible by 10. 22.

45 Ans.

Sol. n is odd number Hence $n = \{11, 13, 15, \dots, 99\}$ Number of values of 'n' is 45

23. Find the value of :
$$\int_{-2}^{2} |(x-2)(x+1)| dx$$

6.33 Ans.

Sol.
$$I = \int_{0}^{2} |(x-2)(x+1)| + |(-x-2)(-x+1)| dx$$

 $= \int_{0}^{2} |((-x^{2}+x+2)+(x+2)|x-1|) dx$
 $= \left(-\frac{x^{3}}{3} + \frac{x^{2}}{2} + 2x\right)|_{0}^{2} + \int_{0}^{1} (x+2)(-x+1) dx + \int_{1}^{2} (x^{2}+x-2) dx$





$$= \left(-\frac{8}{3}+2+4\right) + \int_{0}^{1} \left(-x^{2}-x+2\right) dx + \left(\frac{x^{3}}{3}+\frac{x^{2}}{2}-2x\right)\Big|_{1}^{2}$$
$$= \frac{10}{3} + \left(-\frac{x^{3}}{3}-\frac{x^{2}}{2}+2x\right)\Big|_{0}^{1} + \left(\frac{8}{3}+2-4\right) - \left(\frac{1}{3}+\frac{1}{2}-2\right)$$
$$= \frac{10}{3} + \left(-\frac{1}{3}-\frac{1}{2}+2\right) + \frac{2}{3} + \frac{7}{6} = \frac{19}{3}$$

- 24. Out of '400' people 160 are nonveg and smoker, 140 are veg and smoker and 100 are veg and non smokers. The percentage of people suffering from a certain chest disease in 35%, 20% and 10% respectively. If a person suffers from this disease then find the probability that he is non veg and smoker.
- **Ans.** 0.5957
- Sol. Nonveg + smoker $\frac{160}{400} \xrightarrow{\text{disease}} \frac{160}{400} \times \frac{35}{100}$ Veg + smoker $\frac{140}{400} \xrightarrow{\text{disease}} \frac{140}{400} \times \frac{20}{100}$ Veg + smoker $\frac{100}{400} \xrightarrow{\text{disease}} \frac{100}{400} \times \frac{10}{100}$ Required probability = $\frac{\frac{160 \times 35}{400 \times 100}}{\frac{160 \times 35}{400 \times 100} + \frac{140 \times 20}{400 \times 100} + \frac{100 \times 10}{400 \times 100}}$ = $\frac{16 \times 35}{16 \times 35 + 14 \times 20 + 100} = \frac{560}{940} = \frac{56}{94} = \frac{28}{47} = 0.5957$
- 25. If $\bar{a}=3\hat{i}+\alpha\hat{j}+k$, $\bar{b}=\hat{i}-\alpha\hat{j}+3\hat{k}$ and area of parallelogram made by \bar{a} and \bar{b} are adjacent sides is $8\sqrt{3}$ then $\bar{a}.\bar{b}$ is :

Ans. 2

Sol.
$$\overline{a} \times \overline{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & \alpha & 1 \\ 1 & -\alpha & 3 \end{vmatrix} = 4\alpha \hat{i} - 8\hat{j} - 4\alpha \hat{k}$$

area = $|\overline{a} \times \overline{b}| = 8\sqrt{3}$
= $\sqrt{16\alpha^2 + 16\alpha^2 + 64} = 8\sqrt{3}$
 $32\alpha^2 + 64 = 64.3$
 $\alpha^2 + 2 = 2.3 = 6 \Rightarrow a^2 = 4$
 $\alpha = \pm 2$
 $\overline{a}.\overline{b} = 3 - \alpha^2 + 3 = 6 - 4 = 2$





| 26. | If $f(x) =$ | $\begin{cases} \min\{ \mathbf{x} , 4-\mathbf{x}^2 \\ [\mathbf{x}] \end{cases} \end{cases}$ | 2 ² }, | $-3 \le x \le 2$ $2 < x \le 3$ | then numb | ber of | points | of non | differentiability | in |
|-----|-------------|--|-------------------|--------------------------------|-----------|--------|--------|--------|-------------------|----|
|-----|-------------|--|-------------------|--------------------------------|-----------|--------|--------|--------|-------------------|----|

- [-3,3]
- **Ans.** 5
- **Sol.** Using graph of f(x)



